PHY 101: General Physics 1

Department of Physics

NEWTON'S LAWS OF MOTION

1. FORCES

A force is a mechanical effect of the environment on an object. It is either a push or a pull on an object and has both a magnitude (in appropriate units such as newtons, dynes, or pound) and a direction. It can thus be represented by a ***vector***. A force has two basic effects on an object:

1. It can change the motion of the object, which is the subject of Newton's famous second law.
2. It can distort the shape of an object such as by stretching, compressing, or twisting the object.

a. *Types of Forces*

A force can be either due to direct contact (contact force) such as a hand pushing a block or a rope dragging a box or due to influence from afar (action at a distance) such as the gravitational pull of the earth on a satellite or the push of one magnet on another not in contact with it. On the human scale there are many different forces of either type. But on the atomic scale there are only four fundamental forces: gravitational, electromagnetic, weak nuclear, and strong nuclear—all of them actions at a distance.

b. The Resultant of a System of Forces

The vector sum of the forces acting on an object is called the resultant force on the object. The laws of nature are such that when two or more forces are acting at the same point in an object, they can be replaced by their resultant acting at the same point, which will have the same exact effect on the object as the original set of forces.

Problem 1:

In Figure 1 (a), two forces are shown acting at a point in an object. Find the magnitude and direction of the single force that can replace those two forces and have the exact same effect.

Solution:

In the figure below, the resultant R and the replaced forces F_1 and F_2 (in dashed form), as well as F_2 shifted parallel to itself so that it is tail to head with F_1. Since the two original forces are at right angles to each other, we can use the Pythagorean theorem to obtain the magnitude of the resultant force: $\boldsymbol{R}^2 = F^2 + F^2 = (30\text{ lb})^2 + (40\text{ lb})^2 = 2500\text{ lb}^2$. Taking the square root, we obtain $\boldsymbol{R} = 50$ lb. To get the direction of R we determine its angle *6* with the horizontal. We have tan θ= opposite/ adjacent = 40/30 = 1.33 or $\theta = 53°$ Thus R has magnitude 50 lb and acts at an angle 53 ° above the horizontal.

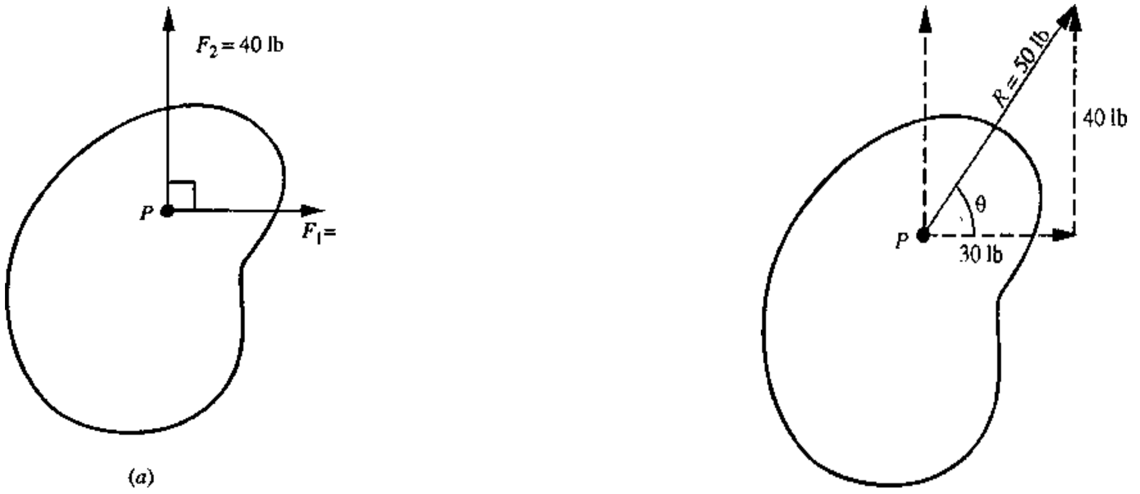


Figure 1

c. *Line of Action*

When a force acts at a point in an object, one can draw an imaginary line through that point and parallel to the force. This is called the **line of action** of the force.

A rigid body refers to an object that doesn't change its shape when forces act on it. No real object is truly rigid, but the concept is a good approximation for stiff objects. In studying the relation of force and motion we will usually assume that we have rigid bodies. While in general the effect of a force on a rigid body depends on where it acts, a force acting on a rigid body can be applied anywhere along its line of action and still have exactly the same effect.

Problem 2.

In figure bellow, we have the same two forces acting on a rigid body as in Figure 1 (a), but now they are acting at different points B and C. Can one still replace these two forces by a single resultant force that has exactly the same effect on the motion of the rigid body and, if so, give an example of such a resultant force?

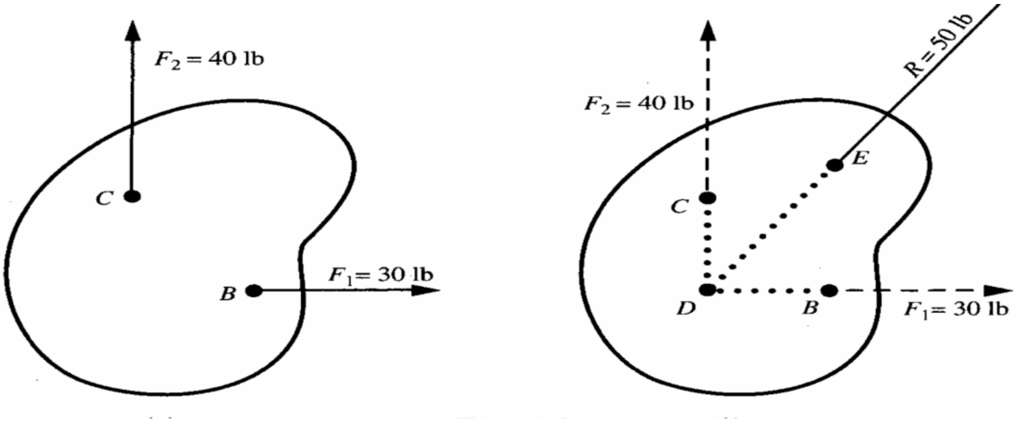


Figure 2

Solution:

The answer is yes. Since F_1 and F_2 can be moved anywhere along their lines of action without changing their effects, we can imagine moving them so that they both act at point D, the intersection of their lines of action (Figure 2(b)). They can then be replaced by their resultant R, acting at the same point D. As already calculated in Problem 1, R is 50 lb acting 53 ° above the horizontal. Furthermore, this resultant force can be moved or slid anywhere along its own line of action without change in effect. Figure (b) shows the resultant R acting at point E, where it still has exactly the same effect as the original two forces (shown in dashed form) that it has replaced.

2. EQUILIBRIUM

Translational motion is the motion of the object *as a whole* through space, without regard to how it spins on itself. The translational motion of a very small object, idealized as a particle, is just the motion of the particle along its path. For a large, irregular body it is less clear what is meant by the motion of the object as a whole or the path of the object through space. Fortunately, the idea can still be defined precisely as the motion of a special point of the object, called the **center of mass.** For simple uniform symmetric objects, such as a disk, a sphere, a rod, or a rectangular solid, the center of mass is at the geometric center of the object.

Problem 3. Describe the translational motion of the board eraser in Figure 3.

Solution

The dashed parabolic line represents the path followed by the center of mass; it thus represents the translational motion of the eraser.

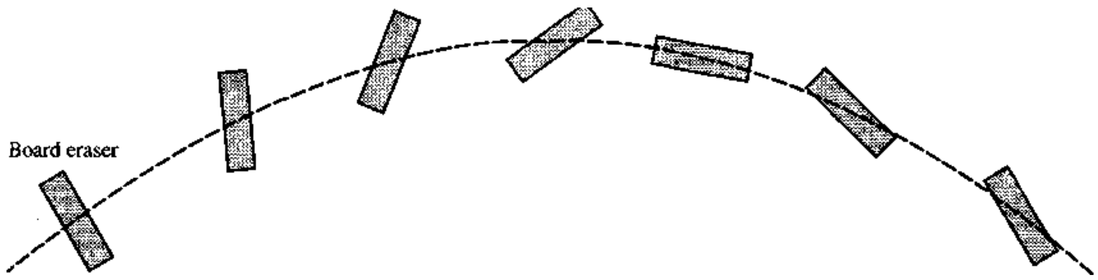


Figure 3

Rotational motion is the spinning motion of an object, without regard to the motion of the object as a whole. Often rotational motion refers to the spinning of an object about a fixed axis, such as the spinning of a wheel on a shaft, but it can also refer to the spinning of an object on itself as the object as a whole moves through space.

Problem 4:

How does one describe the rotational motion of the board eraser from left to right in Figure 3?

Solution

The change in the angular orientation of the eraser represents its rotational motion. Note that the eraser has rotated clockwise through 180 °.

Problem 5:

Describe the translational and rotational motion of the cratered moon around the planet in Fig. 4-4.

Solution

The circular dashed line represents the translational motion of the moon. This moon has no rotational motion since its orientation does not change. The moon, in effect, stays parallel to itself throughout the motion.

Translational equilibrium means that the object as a whole, aside from rotation, has uniform translational motion, that is, its centre of mass is either at rest or moving at constant speed in a straight line.

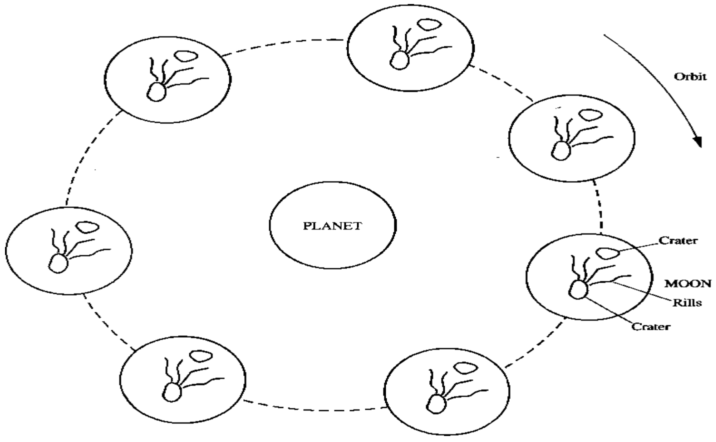


Figure 4

Problem 6: Does the motion of the eraser in Figure 3 or of the moon in Figure 4 correspond to translational equilibrium?

Solution

No. The translational motion of the eraser is a parabolic arc and that of the moon is a circle, whereas for translational equilibrium the motion must be in a straight line. An example of *approximate* translational equilibrium would be a block sliding on an ice-covered lake; the block would move in a straight line without slowing down.

Rotational equilibrium means that the object—whether it is undergoing translational motion or not—is either not spinning or it is spinning in a uniform fashion. For simple symmetric objects this means spinning at a constant rate about a fixed direction.

Problem 7.

Does the motion of the eraser in Figure 3 and of the moon in Figure 4 correspond to rotational equilibrium?

Solution

If the eraser were tumbling at a uniform rate, it would indeed be in rotational equilibrium; that, in fact, is a good approximation to what happens if air resistance is not an important factor. The moon is certainly in rotational equilibrium since we are shown that the moon does not rotate at all.

A Frame of Reference refers to the "framework" that defines the coordinate system in which one's measurements and observations are made. If a coordinate system is fixed to the earth and another one is fixed to a rotating merry-go-round, one is going to observe things differently in each. Each of these coordinate systems is fixed in a different *frame of reference.*

An inertial frame of reference, by definition, is a frame of reference in which a completely isolated object (no forces) will appear to be in both translational and rotational equilibrium. For most purposes the earth can be considered an *inertial frame*; that is only an approximation, however, because the earth spins on its axis—although it is a very slow spin—once every 24 h. The importance of inertial frames is that Newton's laws hold only in such frames, and most of the other laws of physics take on simpler form when described in such frames. We will always assume that we are describing things in an inertial frame of reference unless otherwise indicated.

3. NEWTON'S FIRST LAW

A totally isolated object (no forces) is in both translational and rotational equilibrium in an inertial reference frame. However, even rigid bodies that *do* have forces acting on them can be in either translational or rotational equilibrium, or both, under suitable conditions. The condition for translational equilibrium is the statement of *Newton s first law,* also known as the *law of equilibrium.* We give here some simple cases.

a. *Equilibrium with Only Two Forces Acting*

If the two forces F_1 and F_2 (see Figure 5) are equal in magnitude and opposite in direction (that is, $F_1 + F_2 = 0$), then the object is in translational equilibrium. If in addition the two forces act along a common line of action (collinear forces), as in Figure 5(b), then the object is also in rotational equilibrium.

Note. *It is also possible to have rotational equilibrium without translational equilibrium, a situation that will be discussed in a later chapter.*

Problem 8:

A uniform rod is connected to two cords that exert the only forces on the rod, as depicted in Figure 6; (i.e., we assume there is no pull of gravity on the rod). For each case determine whether the rod is in translational equilibrium. If so, can it also be in rotational equilibrium?

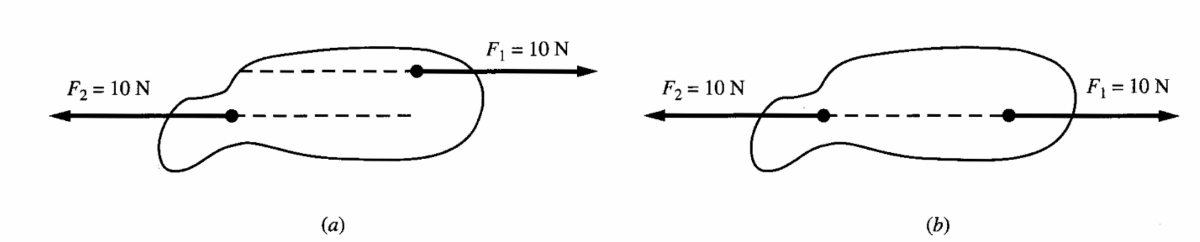


Figure 5

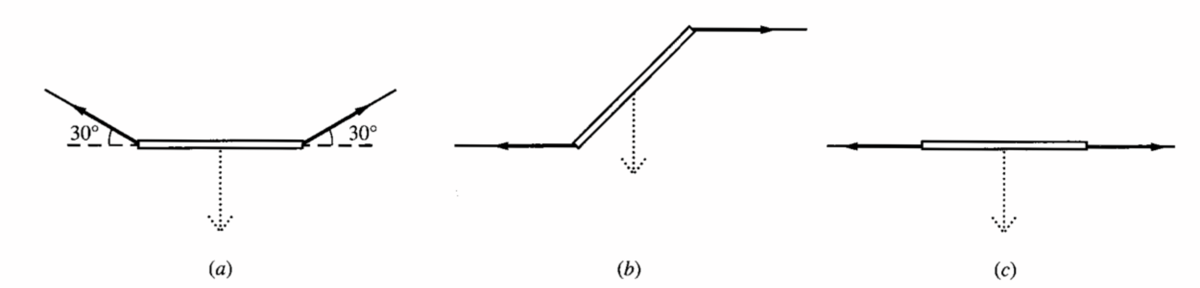


Figure 6

Solution

Since the cords are flexible and exert a force only when they are taut, they can only pull along their length, as is depicted by arrows. Case *(a)* cannot correspond to translational equilibrium because the two forces are not equal and opposite ($F_1 + F_2 \neq 0$), Case *(b)* can correspond to translational equilibrium, *if* the two forces have equal magnitude, but it cannot represent rotational equilibrium because the two forces don't have a common line of action. Case (c) corresponds to both translational and rotational equilibrium if the two cords pull with forces of equal magnitude.

b. *Equilibrium with Three Forces Acting*

If the vector sum of the three forces is zero ($F_1 + F_2 + F_3 = 0$), then the object is in translational equilibrium. If in addition the lines of action of the three forces pass through a common point, then the object is in rotational equilibrium as well. Such a system of forces is called *concurrent.*

Problem 9. Consider the same cases as in Problem 8, except now take into account the weight of the rod. Which of the cases can now correspond to equilibrium?

Solution

Since the rod is uniform, we can assume the weight is a single force acting downward at its center (dotted arrows in Figure 6). Now only case *(a)* can correspond to translational equilibrium since only in that case could the vector sum of the three forces add up to zero if the magnitudes were suitable (see Problem 10). The rod would also be in rotational equilibrium, because, by symmetry, the three forces are concurrent. In neither case *(b)* nor (c) could the three vector forces add up to zero since the weight is perpendicular to the vector sum of the two other forces and could never be balanced by them.

Problem 10. For case *(a)* of Problem 9, if the weight is 100 N, find the force exerted on the rod by each of the two cords if the rod is in equilibrium *(a)* by geometric means; (*b*) by the component method.

Solution

a. Newton's first law tells us that the resultant of the three forces acting on the rod must be zero. In Fig. 4-7*(a)* we redraw the rod as an isolated object and include only the forces acting on it (body diagram). The condition $\mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3} = \mathbf{0}$ implies that the three vectors, drawn head to tail, form a closed triangle. As can be seen in Fig. 4-7*(b),* the triangle is equilateral for our case, so F_1=F_2 = F_2 = 100 N.

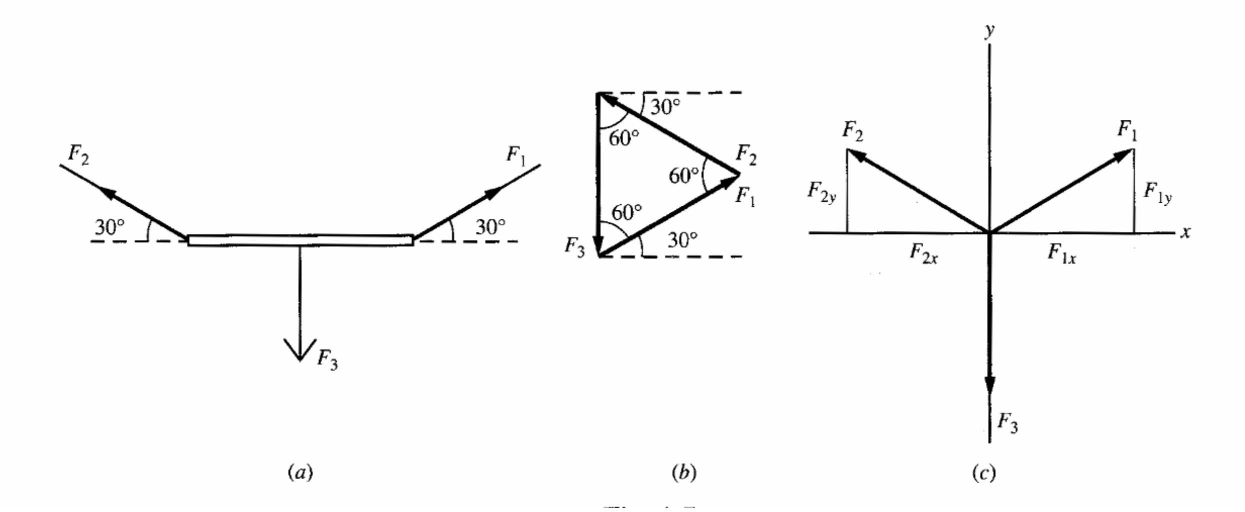


Figure 7

b. We now solve the problem algebraically. Choose the *x* axis along the rod and the *y* axis perpendicular to the rod at its center. Now slide the vectors parallel to themselves to the origin, for easier visualization Figure 7(c). Since the vector sum of the three forces equals zero, we must have for the components.

$$F_{1x} + F_{2x} + F_{3x} = 0 \qquad \text{and} \qquad F_{1y} + F_{2y} + F_{3y} = 0$$

From Figure 7(c), we have

$$F_{1x} = F_1 \cos 30° \qquad F_{2x} = - F_2 \cos 30° \qquad F_{3x} = 0$$

$$F_{1y} = F_1 \sin 30° \qquad F_{2y} = F_2 \sin 30° \qquad F_{3y} = 100N$$

Substituting into the *x*-component equation,

$$F_1 \cos 30° - F_2 \cos 30° + 0 = 0 \qquad \text{or} \qquad F_1 = F_2$$

Similarly, the y-component equation gives.

$$F_1 \sin 30° + F_2 \sin 30° - 100 = 0 \qquad \text{or} \qquad 0.5F_1 + 0.5F_2 = 100 \text{ N}$$

Using $F_1 = F_2$ in the y-component equation gives

$$0.5F_1 + 0.5F_2 = 100 \text{ N} \qquad \text{or} \qquad F_1 = 100 \text{ N} = F_2$$

While this method of solving a vector equation seems more cumbersome than the geometric method, it can be applied to more general cases where die geometric approach is too difficult to use.

c. *Equilibrium with Any Number of Forces*

For the general case of any number n of forces, we again have two conditions for equilibrium. The first is the condition for translational equilibrium, or *Newton s first law,* which says that the vector sum of all the forces is zero: $\sum F_i = 0$. For small objects or particles, where rotation can be ignored, it is the only condition of equilibrium. For extended objects, the second condition, for rotational equilibrium, is again needed. The general case of rotational equilibrium will be discussed in a later chapter. The rest of this chapter is concerned only with translational equilibrium.

4. NEWTON'S THIRD LAW

This law, otherwise known as the *law of action and reaction*, states that if some object *A* exerts a force $\mathbf{F_{ab}}$ on object *B,* then object *B* exerts a force $\mathbf{F_{ba}}$ on object *A* that is equal in magnitude and opposite in direction: $F_{ba} = -F_{ab}$ The law holds both for contact forces and for action-at-a-distance forces.

Problem 11. Consider a book lying at rest on a horizontal table.

(a) What are the forces on the book?

(b) What is the reaction force to each of these forces?

(c) What effect do the reaction forces have on the book?

Solution

(a) There are two forces acting on the book: its weight (the downward pull of gravity toward the center of the earth) and the force exerted upward on the book by the tabletop.

(b) The reaction to the weight is an upward pull of equal magnitude exerted on the earth by the book. The reaction to the table's force is a downward push of equal magnitude on the table by the book.

(c) The reaction forces have no effect on the book! By definition, any effect on the book is represented by a force *on the book.* The reaction forces act on the earth and on the table—not on the book.

Problem 12. An elephant and a teenager are having a tug-of-war, as shown in Fig. 4-8(a). Does Newton's third law imply a draw?

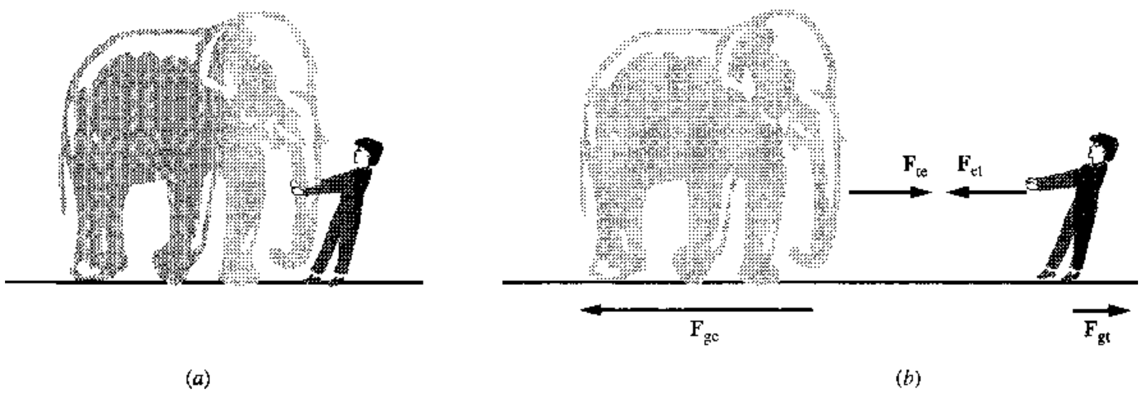


Figure 8

Solution

No. Unless the elephant is very weak, the teenager will definitely lose. It is true that the force the elephant exerts on the teenager $\mathbf{F_{et}}$ is equal and opposite to the force the teenager exerts on the elephant $\mathbf{F_{te}}$, but the motion of either "object" depends on the resultant of *all* the forces acting on it. Both the teenager and the elephant are pushing the ground forward with their feet, and in each case the ground exerts an opposite reaction force. The situation is depicted in Figure 8(b), where $\mathbf{F_{gt}}$ and $\mathbf{F_{ge}}$ represent the horizontal forces exerted by the ground on the teenager and on the elephant, respectively. Thus, for example, suppose that $F_{et} = F_{te} = 250$ lb. We might have $F_{gt}= 100$ lb and $F_{ge} =$ 650 lb. Then a net force of 150 lb acts on the teenager to the left, and he moves leftward. Similarly, a net force of 400 lb acts on the elephant to the left, and the elephant also moves leftward. The next section deals with friction and shows why it is reasonable to assume that $F_{ge}>F_{gt}$.

a. *Tension*

At any given point in a taut rope (or cord, string, thread, or cable) we can ask: With what force does the segment of rope on one side of the point pull on the segment of rope on the other side? Consider the situation in Figure 9(a), where a girl pulls on one end of a horizontal rope with a force **F,** while the other end is attached to the wall. We consider an arbitrary point *p* of the rope that divides it into two segments *A* and *B,* as shown. Figure 9(b) shows the segments as separate bodies, with the horizontal forces on each drawn in. By Newton's third law, the forces with which the two segments pull on each other $\mathbf{F_{ab}}$ and $\mathbf{F_{ba}}$ are equal in magnitude and opposite in direction. The **tension** *T* at the point *p* is the magnitude of either of these forces: $T= F_{ab} = F_{ba}$. Since each rope segment is in equilibrium, we also have $F_{ab} = F$, and $F_w = F_{ba}$, where $\mathbf{F_w}$ is the force of the wall on the rope. Thus, all these forces have the same magnitude *T.* Furthermore, since point *p* was chosen arbitrarily, we conclude that the tension is the same everywhere in the rope.

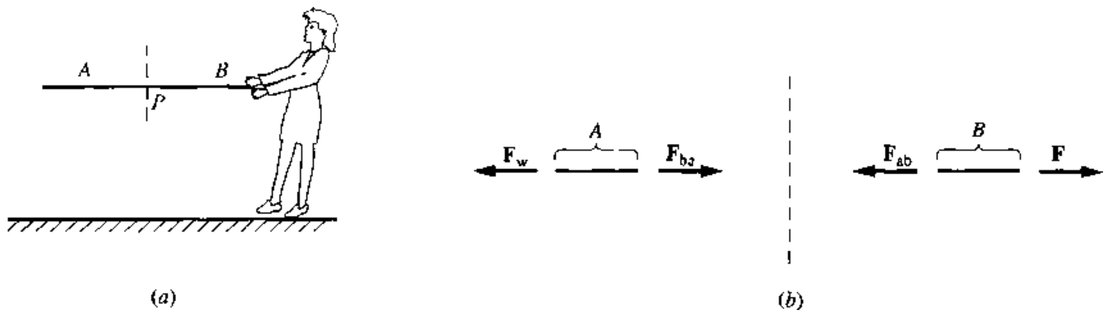


Figure 9

b. *"Weightless" Ropes*

In general, these results are true only for a horizontal rope in equilibrium. If the rope were vertical, with one end attached to the ceiling and the other end pulled down by the girl, then the weight of each segment of the rope would have to be taken into account, and the tension at a point p of the rope would equal neither the force with which the girl pulled down nor the force with which the ceiling pulled up. Indeed, the tension would vary from point to point in the rope. The same would be true if we had a horizontal rope that was not in equilibrium, because the forces applied to either end would not balance out.

There is, however, one circumstance where there is a common tension throughout the rope, and this tension always equals the magnitude of the forces acting at the ends of the rope—whether the rope is horizontal or vertical, whether the rope is in equilibrium or not. This is the circumstance where the rope is weightless. In most problems one characterizes such a rope as a cord, string, or thread to indicate its "lightness." Obviously, no cord is completely weightless, but if it is very light in comparison to the other objects in the problem, it can be assumed weightless without much error.

Problem 13. A block of weight w = 15 N hangs at the end of a (weightless) cord suspended from the ceiling. What is the tension in the cord, and with what force does the cord pull down on the ceiling?

Solution

The tension is the same at all points of the cord and is equal to the magnitude of the force pulling at either end. Since the block is in equilibrium under the action of two vertical forces (the weight downward and the pull of the cord upward), these two forces must have the same magnitude. Hence the upward pull of the cord = 15 N. By Newton's third law the magnitude of the pull of the block downward on the cord is also 15 N, so T - w = 15 N. The tension T also equals the magnitude of the pull of the ceiling on the cord, which by Newton's third law equals the pull of the cord downward on the ceiling. Thus, the downward pull of the top of the cord on the ceiling is the same as the downward pull of the block on the bottom of the cord. Thus, we see that a weightless rope transmits an applied force from one end to the other.

5. FRICTION

Friction is the rubbing force between two objects whose surfaces are in contact. The force of friction always acts parallel to the touching surfaces. By Newton's third law each surface exerts a frictional force that is equal in magnitude and opposite in direction to that exerted by the other. The magnitude of the frictional force exerted by each surface on the other depends on how tightly the two surfaces are pressed together.

a. Normal Force

The force responsible for this "pressing together" is called the **normal force** because it acts perpendicular to the two surfaces. By Newton's third law each surface exerts a normal force that is equal in magnitude and opposite in direction to that exerted by the other. Figure 4-10 indicates the frictional and normal force on each object when a block is in contact with an inclined plane. The frictional force (parallel to the surface) and the normal force (perpendicular to the surface) acting on a surface can always be thought of as the components of the overall force acting on that surface due to the other surface in contact with it.

b. Static Friction

When two surfaces are at rest with respect to one another, the frictional force each exerts on the other always opposes any tendency to relative motion. The frictional force on an object adjusts itself in magnitude and direction to oppose and counterbalance any other forces on the object that would tend to make the object start to slide. It varies, as needed, from zero magnitude up to some maximum value to stop such slippage. Such a frictional force is called a **static friction** force (f_s). The maximum

Figure 10

static friction force $f_{s,max}$ that one surface can exert on another is proportional to the normal force N between the surfaces: $f_{s,max} = \mu_s N$, where N is the magnitude of the normal force, and n_s is a proportionally constant, called the **coefficient of static friction,** that depends on the nature of the two surfaces. It is possible to force one object to slide over the other by applying a parallel force to one of the objects that is larger than ($\mu_s N$, the maximum possible static friction force.

Problem 14: A book of weight $w = 10$ N rests on a horizontal tabletop, as shown in Figure 11(a), and a horizontal force F is applied to it. If the coefficient of static friction $\mu_s N$ between the book and the tabletop is 0.25, calculate *(a)* the normal force exerted by the tabletop on the book, and (b) the maximum value of the static friction force.

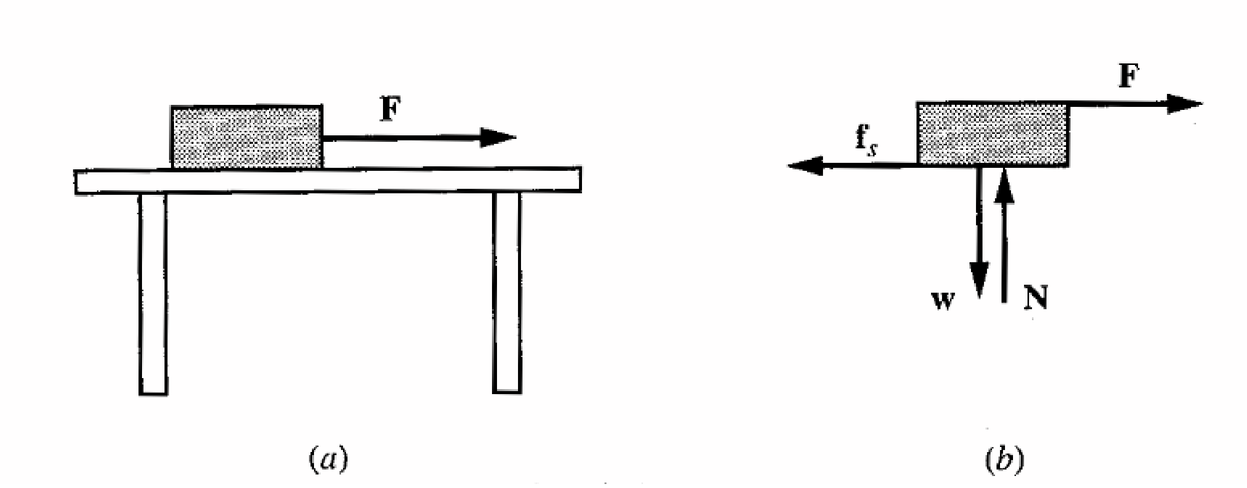


Figure 11

Solution

(a) Since the book is in equilibrium, the sum of the forces acting on it must equal zero. Figure 11*(b)* shows the body diagram for the book with all the forces acting on it. The frictional force is $\boldsymbol{f_s}$, and the normal force is **N**. Noting that $\boldsymbol{f_s}$ and **F** have no y components, from the condition that the sum of the y components equals zero we have $N - 10N = 0$, or $N = 10N$.

(b) The maximum value attainable by the static friction force is.

$$f_{s,max} = \mu_s N = (0.25)\,(10N) = 2.5N$$

Problem 15.

(a) In Problem 14, if the magnitude of the applied force is $F = 2.0N$, what is the magnitude and direction of the frictional force on the book?

(b) What if $F = 1.0N$; $0N$?

(c) What is the biggest value that F can be before the book starts to slide?

Solution

(a) The frictional force opposes the tendency to motion, so it is in the direction opposite to F, as shown in Figure 11(b). The magnitude of the frictional force adjusts itself to keep the book at rest, which in this case means $f_s = F = 2.0$ N. This value is possible, since it is smaller than the maximum found in Problem 14(b).

(b) If $F = 1.0$ N, then, by the same reasoning as in part (a), we have $f_s = 1.0$ N in the direction *opposite to* F. If $F = 0$, then $f_s = 0$, and there is no frictional force at all.

(c) If F is bigger than $f_{s,max}$, then the frictional force cannot rise to match F and maintain equilibrium. Thus $F = 2.5N$ is the limiting value; beyond this value equilibrium cannot be maintained, and the book starts to move.

c. *Kinetic Friction*

Once two surfaces are in motion relative to one another, the frictional force, now called **kinetic friction (f_k)**, acting on a surface is always in a direction opposed to the velocity of that surface. To a good approximation, its magnitude is independent of the magnitude of the velocity and is again proportional to the normal force between the two surfaces. Thus, it can be expressed as $f_k = \mu_k N$, where μ_k, the **coefficient of kinetic friction**, depends only on the nature of the two surfaces. For any given pair of surfaces, $\mu_k > \mu_s$.

Problem 16. Assume the book in Figure 11(a) is moving to the right with speed v.

(a) Now what are the magnitude and direction of the force of friction exerted by the tabletop on the book?

(b) Does f_k depend on the magnitude of the applied force F?

(c) If the book instead moves to the left with speed v, with **F** still to the right, what are the magnitude and direction of the force of friction? Assume that $\mu_k = 0.2$.

Solution

(a) Once the book is moving the (kinetic) friction is of fixed magnitude, $f_k = \mu_k N$. Since we still have equilibrium in, they direction, we still have the same normal force; Thus $f_k = (0.2)\ (10\ \text{N}) = 2.0$ N. The direction of the kinetic friction force is always opposite to the direction of motion, so it would be to the left.

(b) No.

(c) Since the normal force is still the same, the value *off* is still 2.0 N. The direction of f* is now to the right. Note that the direction of f* depends only on the direction of motion and not on the direction of **F**.

6. CORDS AND PULLEYS

If a (weightless) cord is bent over a pulley, as in Figure 12, there are two idealized situations in which the tension in the part of the cord on one side of the pulley will be the same as the tension in the part of the cord on the other side of the pulley.

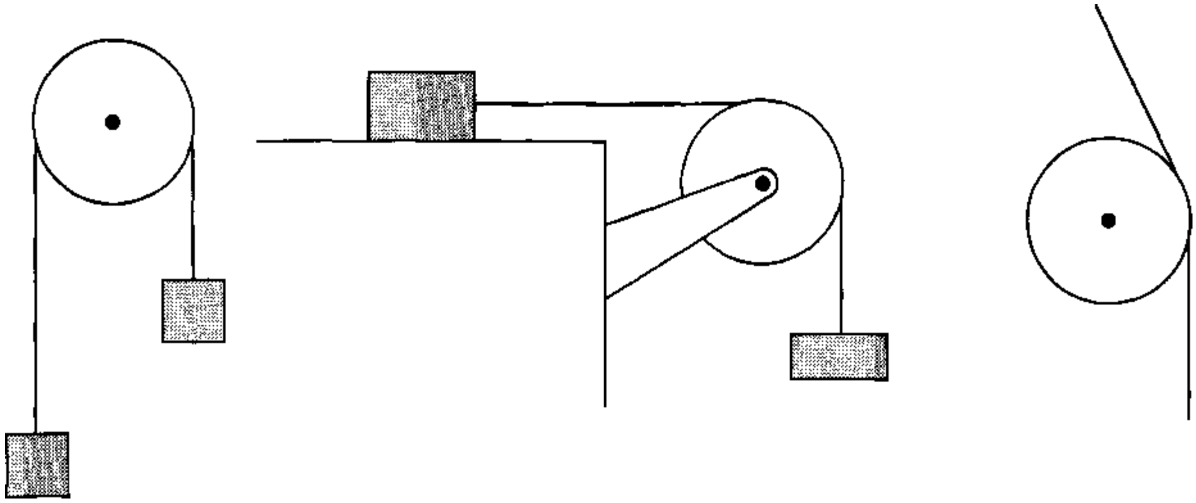


Figure 12

1. The surface of the pulley is frictionless so that the cord slides effortlessly over it (frictionless pulley).

2. The surface of the pulley has friction, *but* the pulley has no weight *and* there is no friction between the pulley and the axle on which it rotates (weightless pulley).

In a problem, being told that the pulley is frictionless and/or weightless (massless) is generally shorthand for case 1 or case 2, and you can assume as much unless told otherwise.

Problem 17: In Figure 13(a), the two blocks are connected by a light rope over a frictionless, weightless pulley. If the system is initially at rest, will it stay at rest? If so, what is the frictional force exerted by the table on block *A?*

Solution

Figure 4-13(b) gives the body diagrams for the two blocks. For block *B,* assuming equilibrium, the ^-component equation gives $T - W_b = 0$ or $T = W_b = 10\ N$. Since we have a rope and a frictionless, weightless pulley, the tension is the same on the blocks side of the pulley, and T=10N for block A as well.

Vertical equilibrium of block A requires that $N—W_a=0$, or $N=W_a=30$N. Then the maximum possible static frictional force is

$$f_{s,max} = \mu_s N(0.5)\,(30N) \ = \ 15N$$

Since $T<f_{S,\max}$, the frictional force can balance T and the system remains at rest. The actual frictional force can be obtained from the equilibrium of block *A:*

$$T—f_s—0 \qquad \text{or} \qquad f_s=T=10\text{N}$$

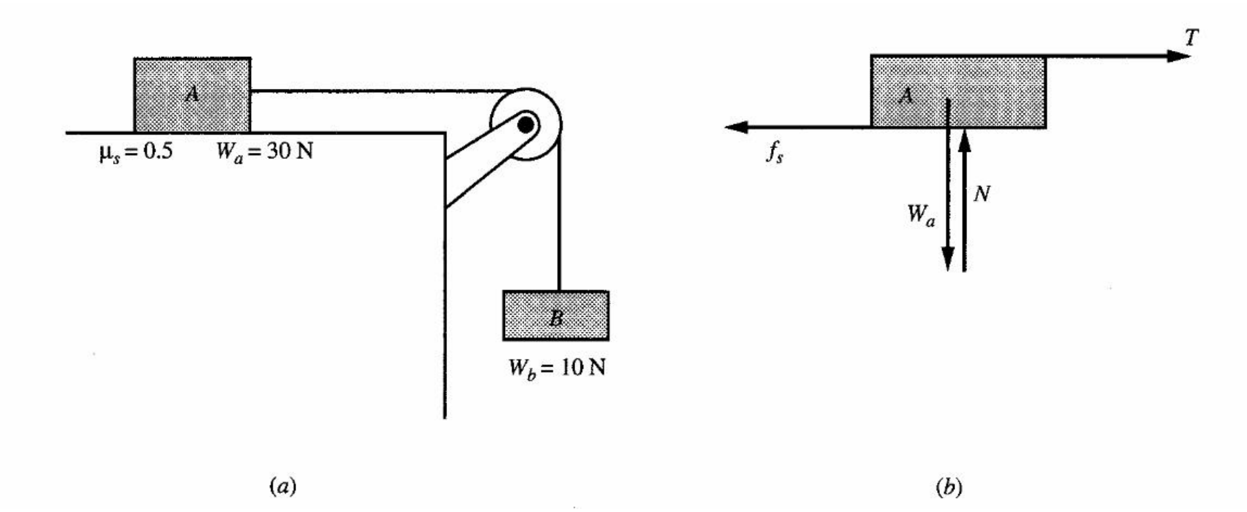


Figure 13

Problem 18: Find the resultant **R** of the two forces shown in Figure 14.

Solution

R=F_1 + F_2. We choose *x* and *y* axes as shown in the figure and use the component method of addition.

$$F_{1x} = 0 \qquad F_{1y} = 20N \qquad F_{2x} = -(60N)\cos 37° \qquad F_{2y} = -(60N)\sin 37°$$

$$R_x = F_{1x} + F_{2x} = 0 - (60N)(0.8) = -48N$$

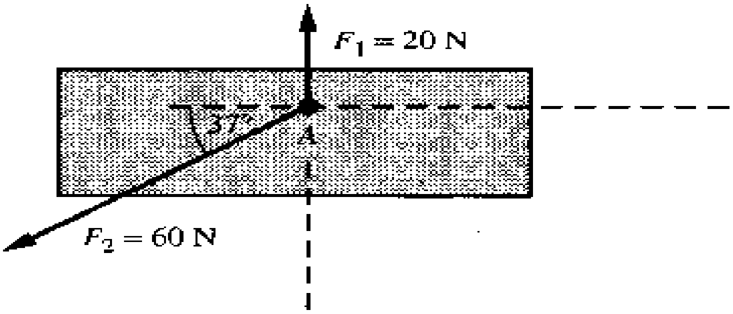
$$R_y = F_{1y} + F_{2y} = (20N) - (60N)(0.6) = -16N$$

$$R = [(-48)^2 + (-16)^2]^{\frac{1}{2}} = 50.6N$$

From the signs of its components, R is in the third quadrant. If 6 is the acute angle that R makes with the negative *x* axis,

$$\tan\theta = \left|\frac{R_y}{R_x}\right| = \frac{16}{48} = \frac{1}{3} \qquad \text{or} \qquad \theta = 18.4°$$

Thus, **R** has magnitude 50.6 N and points away from the origin at 18.4° below the negative *x* axis.



Problem 19. Three forces act on a rigid body, as shown in Figure 15, with their lines of action passing through the common point ***B.*** Find their resultant and its point of application for equilibrium.

Solution

$\boldsymbol{R = F_1 + F_2 + F_3}$. Choose the *x* and *y* axes as shown. Then

$$R_x = F_{1x} + F_{2x} + F_{3x} = (-50\text{ N})\cos 30° + (40\text{N})\cos 45° + (0\text{N})$$

$$= (-50N)(0.866) + (40N)(0.707) = -15.0N$$

$$R_y = F_{1y} + F_{2y} + F_{3y} = (50N)sin30° + (40N)sin\,45\,° + (-30N)$$

$$= (50N)\,(0.5) + (40N)\,(0.707) + (-30N) = 23.3N$$

$$R = [(-15)^2 + (23.3)^2]^{\frac{1}{2}} = 27.7\,N$$

R is in the second quadrant, with

$\tan\theta = \left|\frac{R_y}{R_x}\right| = \frac{23.3}{15.0}$ or $\theta = 57.2\,°$ above the negative x axis

R can act anywhere along a line of action through *B.*

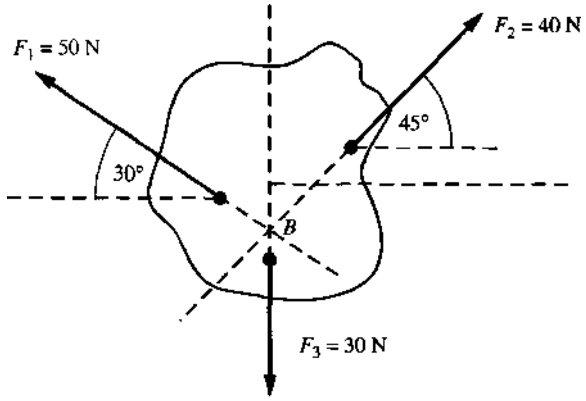


Figure 15

Problem 20: Refer to Problem 18.

(a) What third force E, must be exerted on the body for it to be in translational equilibrium?

(b) Where must E be applied to give rotational equilibrium as well?

Solution

(a) For translational equilibrium, $\boldsymbol{F_1} + \boldsymbol{F_2} + \boldsymbol{E} = \boldsymbol{0}$, or $E = -(F_1 + F_2) = -R$. Hence $E = 50.6\,N$, and E points 18.4° above the positive x axis (see ***Figure 16***).

(b) ***E*** must have the same line of action as R; that is, its line of action must also pass-through point *A.*

Note. The force which, when added to an existing set of forces on an object, will cause the object to be in equilibrium is called the *equilibrant* of the set. (The force E in the previous problem is thus an equilibrant.)

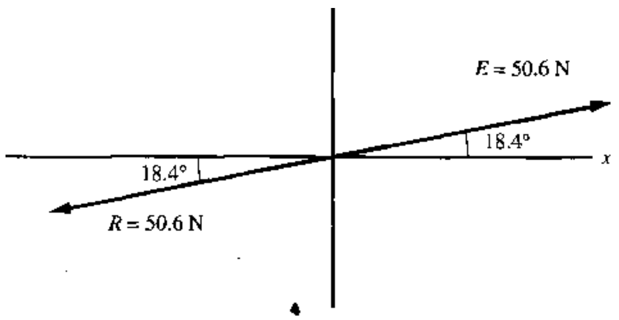


Figure 16

Problem 21. Find the equilibrant of the forces in Problem19.

Solution

Here we have the concurrent forces **F₁, F₂,** and **F₃** which can be replaced by the single resultant force R = **F₁ + F₂ + F₃** with line of action through point **B**, as obtained in Problem 19. Clearly, to have equilibrium, the added fourth force, the equilibrant E, must obey $E = -R$. Thus $E = 27.7\ N$ pointing 57.2° below the positive *x* axis, with a line of action that must also pass through point *B.*

Problem 4.22. A block of weight = 400 N hangs from a uniform heavy rope of length 3 m and weight w_2 = 300 N, as shown in Figure 17(a). Find (a) the force with which the rope pulls on the block; *(b)* the tension in the rope 1 m above the contact point with the block; (c) the force with which the ceiling pulls on the rope.

Solution

In Figure 17*(b)* we have the body diagrams for the block, the lower third of the rope, and the full rope, respectively. Each is in equilibrium, and the vector sum of the forces on each equal zero. Since the forces are all in the *y* direction, only the equilibrium condition in that direction need be applied.

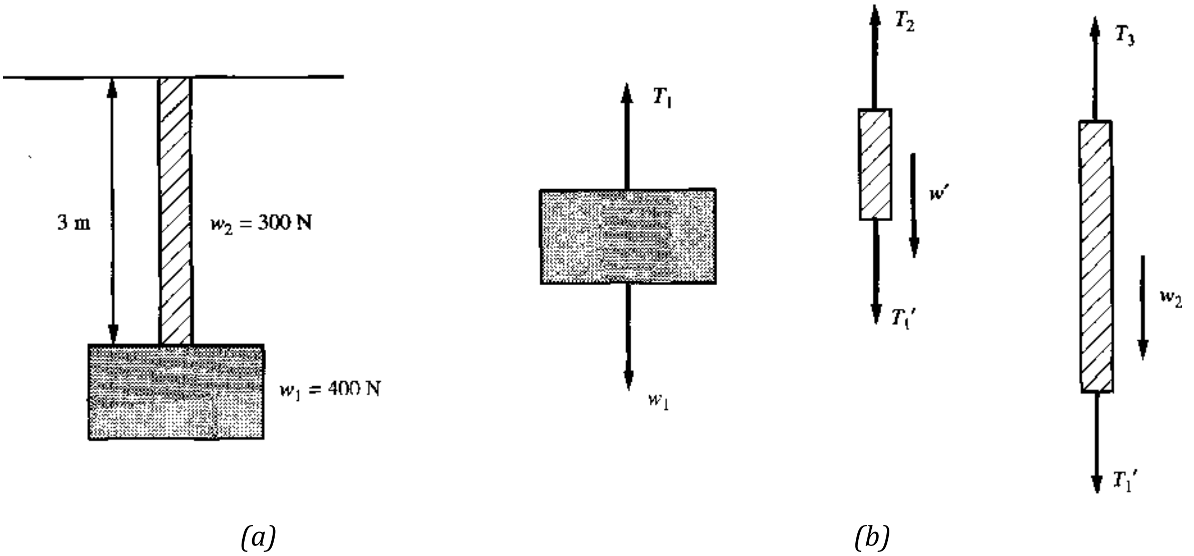


Figure 17

(a) For the block, $T_1 - w_1 = 0$, or $T_1 = 400\ N$ equals the force of the rope on the block.

(b) For the lower third of the rope, $T_2 - T_1' - w' = 0$, where T_2 is the contact force of the upper two- thirds of the rope on the lower third and is the tension in the rope at that point; *T[* is the force of the block on the rope, given by Newton's third law as $T_1' = T_1 = 400\ N$; $w'$is the weight of the lower third of the rope, or $w' = 100N'$ Thus $=\ 400\ N\ +\ 100\ N\ =\ 500\ N.$

(c) For the rope as a whole,$T_3 - T_1' - w_2 = 0$, or $T_3 =\ T_1' + w_2 = 400N + 300\ N = 700\ N,$ equals the force of the ceiling on the rope.

Problem 23: For the weight-and-strings setup of Fig.18(a), find the tensions T_1 T_2, and T_3.

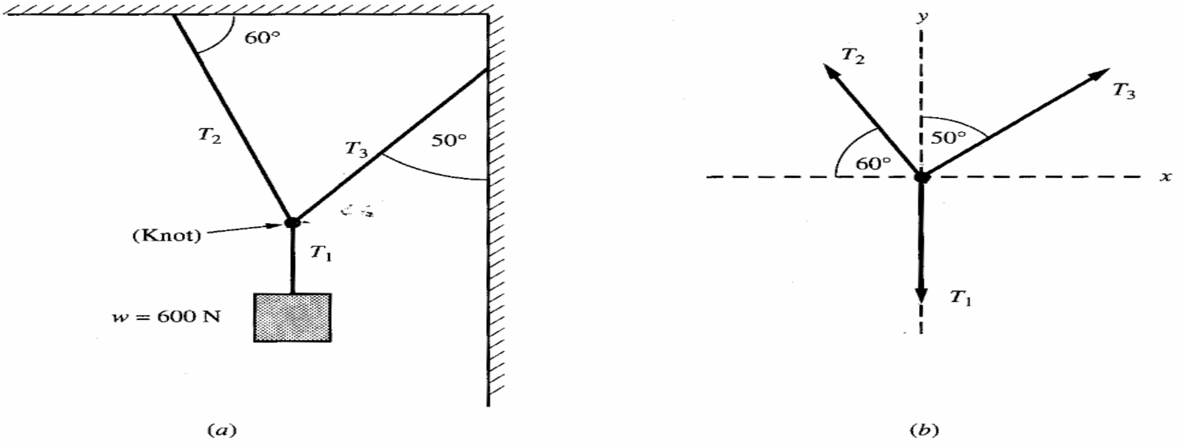


Figure 18

Solution

From the equilibrium of the block, T_1= 600 N. Since the knot is in equilibrium, the body diagram, Figure18(b), gives $T_1 + T_2 + T_3 = 0$. Using the component method, we get

$T_{1x} + T_{2x}T_{3x} = 0 - T_2 \cos 60° + T_2 \sin 50° = 0$ or $0.5T_2 = 0.766T_3$ or $T_2 = 1.532\ T_3$.

(A sine appears in the x-component equation because the angle of T_3 is given relative to the *y* axis). Similarly,

$\boldsymbol{T_{1y}} + \boldsymbol{T_{2y}} + T_{3y} = -\boldsymbol{T_1} + \boldsymbol{T_2}sin60° + T_3\ cos50° = 0$ or $0.866T_2 + 0.643T_3 = 600N.$

Substituting for T_2,

$(0.866)\ (1.532T_3) + 0.643T_3 = 600N$ or $1.97073 = 600N$ or $T_3 = 305N$

Finally, $T_2 = 1.532T_3 = 467$ N.

Problem 24: A block of weight w = 200 N is pulled along a horizontal surface at constant speed by a force F = .80 N acting at an angle of 30° above the horizontal, as shown in Figure19.

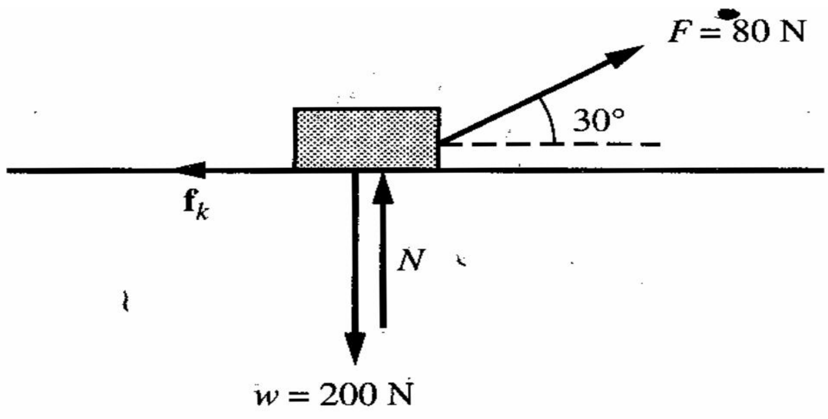


Figure19

(a) Find the frictional force **f** exerted on the block by the surface.

(b) Find the normal force **N** exerted on the block by the surface.

(c) Find the coefficient of kinetic friction, *fi*$_k$, between the block and the surface.

Solution

(a) The four vector forces acting on the block are shown in Fig. 4-19. Since the block is in equilibrium; their sum equals zero. For the *x* components we thus have

$$F\cos 30^0 - f_k = 0 \qquad \text{or} \qquad f_k = (80\ \text{N})\,(0.866) = 69.3\ \text{N}$$

(*b*) Similarly, for the *y* components,

$$F\sin 30° + N - w = 0 \qquad \text{or} \qquad N = 200\text{N} - (80\text{N})\,(0.5) = 160\text{N}$$

Note that the normal force is not equal to the weight even though the block is on a horizontal surface, because the force **F** has a vertical component.

(c) $\mu_k = f_k/N = 69.3/160 = 0.433.$

Problem 25: A hanging weight w_1 is connected by a light cord over a frictionless pulley to a block on a frictionless incline of weight w_2=500N, as shown in *Figure 20*. If the block on the incline moves down at constant speed, what is the weight of the hanging block? How would your answer change if it were moving up the incline at constant speed?

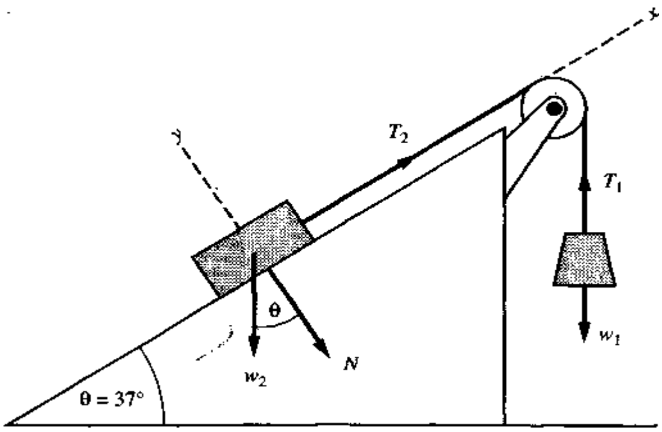


Figure 20

Solution

In Figure20, all the forces on the respective blocks are shown right on the diagram for the system as a whole. Since both blocks move in straight lines at constant speed, they are each in equilibrium. For the hanging block, using y components, we have $T_1 — w_1 = 0$, or w_1=T_1. To find T_1 we turn to the block on the incline. We choose x and *y* axes along the incline and perpendicular to it, respectively. We also note that the force of the cord on each block has the same magnitude, so T_2-T=T, since the cord is light, and the pulley is frictionless. Then, for the x-component equilibrium equation we get

$$T - w_2 \sin\theta = 0 \qquad \text{or} \qquad T = (500\ N)\,(\sin 37°) = 300N$$

Then from our earlier result $w_1 = T = 300$ N. Note that we did not need to solve the y-component equilibrium equation for the block on the incline to solve for *T* and w_1. This is because the y-component equation gives us the normal force *N*., which does not affect the *x-component* equation when there is no friction. If the block were moving up the incline, the blocks would still be in equilibrium under the action of the same forces, so the answer would remain the same.

Problem 26. Suppose that in Problem 25 there was friction between the block and the incline, and that the coefficient of sliding friction was $\mu_k = 0.3$, but all the other data in the problem remained unchanged. Find the weight of the hanging block, w_1 if the other block moves at constant speed *(a)*down the incline; *(b)* up the incline.

Solution

(a) We can use Figure 20 with the modification that there is an additional force on the block on the incline, a frictional force of magnitude f_k opposing the motion of the block and hence pointing parallel to the incline in the upward direction. From the rules for friction, we have $f_k = \mu_k N$, where ***N*** is the normal force exerted on the block by the incline. Following the reasoning of Problem 25 we now have for the *x components.*

$$T + \mu_k N - w_2 \sin 37° = 0 \text{ or } \qquad T = (500\ N)(0.6) - 0.3\ N$$

For the *y components*

$$N - w_2 \cos 37° = 0 \text{ or } \qquad N = (500N)(0.8) = 400N$$

Substituting into the previous equation we have

$$T = (500\ N)(0.6) - 0.3\,(400\ N) = 300\ N - 120\ N = 180N$$

Since the hanging block obeys $w_1 = T$, we have our result, $w_1 = 180N$.

(b) If the block is moving up the incline at constant speed, we proceed as before, noting that the frictional force is now directed down the incline although it still has the same magnitude $f_k = \mu_k N$. Furthermore, the y-component equation for the block on the incline is unchanged, so we still have N = 400 N and f_k = 0.3(400 N) = 120 N. The x-component equation changes only in that the sign of the x-component of the frictional force changes, and we get.

$$T - \mu_k N - w_2 \sin 37° = 0 \qquad \textit{and} \qquad T = 300N + 120N = 420N$$

Finally, $w_1 = T = 420\ N$

Problem 27. For the setup in Figure18(a)—first discussed in Problem 23—the breaking point of the two cords attached to the wall mid ceiling is 1500 N. How heavy can the block be without one of the cords snapping? Assume the cord attached to the block can handle any weight.

Solution

We first determine which of the two cords will reach a tension of 1500 N first. To do this we recall from Problem 23 that equilibrium in the *x* direction requires.

$T_3 \sin 50° = T_2 \cos 60°$ or $0.766T_3 = 0.50T_2$ or $T_3 — 0.653T_2 < T_2$

Clearly T_3 is always less than T_2, and hence T_2 will reach 1500N first. We now set T_2 = 1500 N; from above, this immediately yields T_3 = 0.653 (1500 N) = 980 N. We can now determine the corresponding weight *w* of the block using the equilibrium equation in the *y* direction:

$$w = T_1 = T_2 \sin 60° + T_3 \cos 50° = (1500N)(0.866) + (980N)(0.643) = 1929N$$

7. NEWTON'S SECOND LAW OF MOTION

Newton's second law of motion states that the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass. Mathematically, it is expressed as:

$$F = ma$$

Where:

F is the net force acting on the object (in newtons, N),

m is the mass of the object (in kilograms, kg),

a is the acceleration of the object (in meters per second squared, ms^{-1}).

a. *Resultant force and acceleration*

Earlier it was found that if the vector sum of the forces on an object-the resultant force-is zero, then the object is in translational equilibrium, i.e., it has constant velocity, or, equivalently, zero acceleration. Ifthe resultant force is not zero, then we should expect that the acceleration also would not be zero. Indeed, we should say that the unbalanced force <Yn the object *caused* its acceleration. Newton's second law is the quantitative statement of this cause-and-effect relationship.

i. *Experimental Facts and the Formulation of Newton Second Law*

When a nonzero resultant force F acts on a given object, the consequent acceleration a always points in the direction of F. Also, for a given magnitude of F, the magnitude of a is the same no matter what the direction of the force. On the other hand, if the magnitude of F doubles, the magnitude of a doubles; if the magnitude of F triples, the magnitude of a triples; etc. Thus, the magnitude of a is proportional to the magnitude of F, or $F \propto a$. The proportionality constant is called the *mass m* of the object, and we write $F = ma$, where m is generally different for different objects. Since m is a scalar quantity, we can combine the results for the magnitude and the direction of the acceleration in the single equation.

$$F = ma$$

This equation is the mathematical statement of Newton's second law. In Figure 21 (a) and (b), we show different resultant forces having the same magnitude acting on *(a)* the same object and
(b) different objects, and the resulting accelerations of those objects.

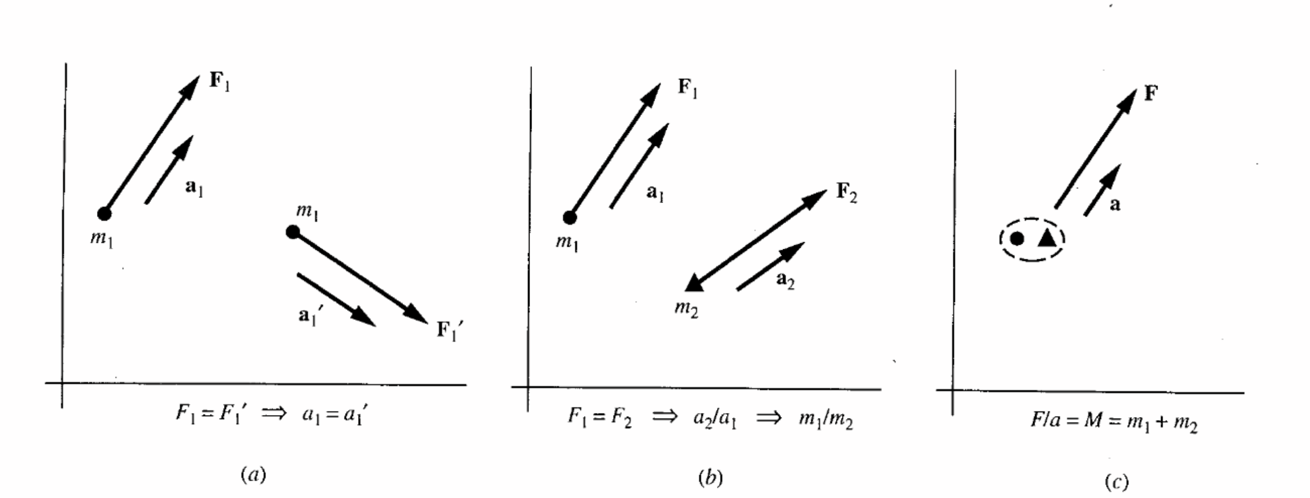


Figure 21

ii. *The Meaning of Mass*

As can be seen in Figure 21 the mass controls the response of the object to a given magnitude force: A small mass means a large acceleration, a large mass means a small acceleration. Because mass measures the resistance of an object to having its velocity changed ("being shoved around"), it is often referred to as the inertia of the object. The relative magnitude of different masses can easily be established by applying the same magnitude force to the different objects and measuring their accelerations. Then

$$m_1 a_1 = m_2 a_2 \qquad \text{or} \qquad \frac{m_1}{m_2} = \frac{a_2}{a_1}$$

The mass is clearly an intrinsic property of an object, but for it to be a truly fundamental property of all matter one needs to show that objects maintain this property even when they are combined with other objects. Figure 5-1(c) shows a resultant force being applied to two objects stuck together. The resulting acceleration is just what one expects if the mass of the combination is $M = m_1 + m_2$. The mass is thus an indestructible and unchanging property of any object that stays with the object even when it is combined into larger units. In the same way, when an object is broken into smaller parts, the sum of the masses of the parts equals the original mass.

iii. Units of Force and Mass

In the International System (SI) units, the unit of force is already determined for us from Newton's second law once we have a unit of mass and a unit of acceleration. The unit of mass is the kilogram, and the unit of acceleration is the meter per second squared. The corresponding unit of force is the Newton (N), and from $F = ma$ we have

$$1N = (1kg)(1ms^{-2}) = 1kg.ms^{-2}$$

In other words, a 1-N force gives a 1-kg mass an acceleration of 1m/s^2. If one chooses the gram as the unit of mass and the centimetre per second squared as the unit of acceleration, then the unit of force is called the dyne. Again, from $F = ma$

$$1\ dyn = (1\ g)(1\ cms^{-2}) = 1\ g.cm\ s^{-2}$$

Problem 5.1: How many dynes are there in a newton?

Solution

1 N = 1 kg·m/s^2 = (1000 g)(1OO cm)/s^2 = 100,000 g cm/s^2 = 10^5 dyn

Problem 28. What is the magnitude and direction of the acceleration of an object whose mass is 10 kg when it is acted on by a resultant force of 380 N at 30° above the positive *x axis*?

Solution

The direction is the same as that of the resultant force, 30° above the positive *x* axis. For the magnitude $F = ma$ gives

$$a = \frac{F}{m} = \frac{380N}{10\ kg} = 38N.kg^{-1} = \frac{38kg\ ms^{-2}}{kg} = 38\ ms^{-2}$$

Problem 29. A constant force acts on a 30-g object and produces an acceleration of 2m/s^2. Find the force in dynes.

Solution

We are given mixed units, so we first convert the acceleration to the gram-centimeter-second system: a = 2 m/s^2 = 200 cm/s^2. Then $F = ma$ gives

$$F = (30\ g)\ (200\ cm/s^2) = 6000\ dyn$$

iv. The English System and Weight

In the English system of units, it is the unit of force, the pound (lb), that is fundamental, rather than the mass. One pound (1 lb) is defined as the pull of gravity on an object whose mass is 0.45359 kg at a specified latitude of the earth's surface. (The pull of gravity on an object is commonly called its weight.) The corresponding unit of mass is now defined using the second law, $F = ma$ mass of 1 slug is that mass which when acted on by a force of 1 lb accelerates at 1 ft/s^2, or 1 slug = (l lb)/(1 ft/s^2 .

To convert from pounds to newtons we have to discuss the nature of weight. If an object near the earth's surface is acted on only by the force of gravity, it will accelerate with the acceleration g = 9.8 m/s^2. Calling the force of gravity, or weight, w, the second law gives $w = mg$. Since g is the same for all objects, $w/m = g$ is constant. Thus, weight and mass are proportional at a given point on the earth's surface. As one changes position on the earth's surface, both w and g vary slightly, but m stays constant. This will be discussed in more detail when we discuss the law of universal gravitation. We can now determine the conversion from the English to the metric system. From its definition: 1 lb = (0.45359 kg)(9.8 m/s^2) = 4.445N. The mass 0.45359 kg is given a special name and called 1 pound-mass (i.e., the mass that weighs 1 lb). Since a force of 1 lb gives 1 lb-mass an acceleration g = 9.8 m/s^2= 32.2 ft/s^2, while it gives 1 slug an acceleration of only 1 ft/s^2, it follows that 1 slug = 32.2 lb-mass = 32.2 (0.45359 kg) = 14.7 kg.

Problem 29 What is the weight w, in pounds, of a 1-kg mass?

Solution

We can first get w in newtons. w = (1 kg)(9.8 m/s^2) = 9.8 N. Dividing by 4.445 N/lb we get w = (9.8.N)/(4.445 N/lb) = 2.20 lb. We could also get the result directly from the fact that 0.45359 kg weighs 1 lb, and therefore 1 kg weighs 1/0.45359 = 2.20 times as much.

Problem 30: A resultant force of 50 lb acts on an object weighing 12 lb. Find the acceleration.

Solution

The mass of the object *is* $m = \frac{w}{g}$ = (12 lb)/(32.2 ft/s^2) = 0.373 slug. Then

$$50 \text{ lb} = (0.373 \text{ slug})a \qquad \text{or} \qquad a = 134 \text{ ft/s}^2$$

8. APPLICATIONS OF THE SECOND LAW

Whenever applying the second law it is essential to clearly identify the object being accelerated and to be sure that the force appearing in the equation is the resultant of all forces acting on the object. Also, because $F = ma$ as a vector equation, it may be useful to resolve it into components along convenient x and y axes.

a. *Forces on a Single Object*

Problem 5.6. A constant force T pulls horizontally on a block of mass m = 2.0 kg, which is free to move on a frictionless horizontal surface, as shown in Figure 22(a). Starting from rest, the block is observed to move 20.0 m in 2.0 s. Find T.

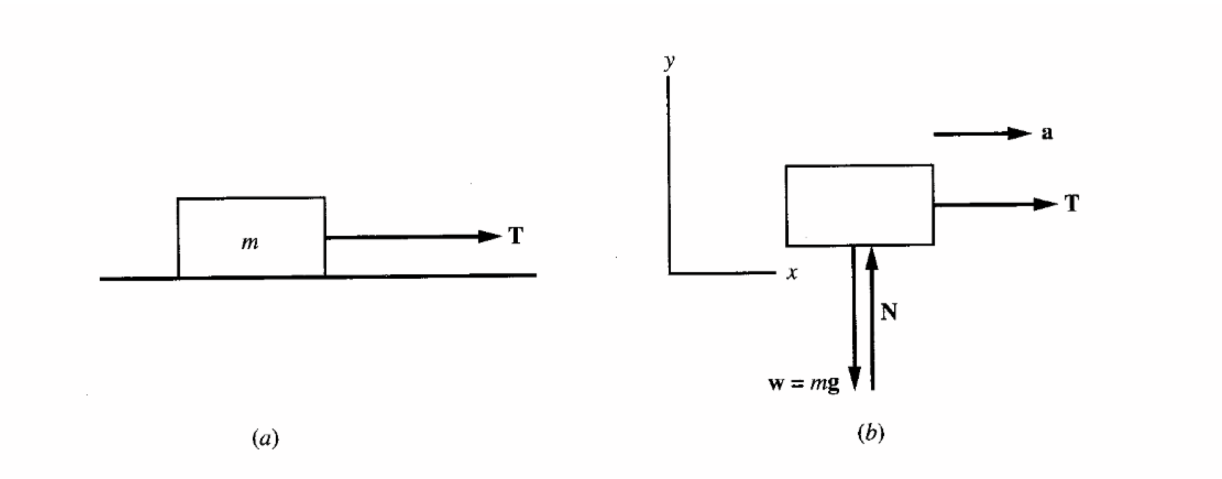


Figure 2

Solution

We first draw a body diagram for the block, with all forces on the block drawn in, as shown in Figure 22*(b).* Also shown is the acceleration a. Since the acceleration is along the *x* direction (the block stays on the table), we have $a_x = a$ and $a_y = 0$. For the *x* direction we have $Fx = max$. Since Tis the only force with an *x* component, and it points in the *x* direction, we have $T = ma$ or $T = (2.0\ kg)a$. Since T is constant, we know that a is constant, and we can use the kinematic equations for constant acceleration, together with the kinematic information given. Since the block starts from rest, we can set $x_0 = v_{0x} = 0$. We then have $x = (1/2)at^2$, which for our case yields 20.0 m = $a\,(2.0\text{s})^2$ or $a = 10.0\ \text{m/s}^2$. Then

$$T = (2.0\ \text{kg})\ (10.0\ \text{m/s}^2) = 20.0\ \text{N}$$

Problem 31. Redo Problem 30, if there is now friction between the block and tabletop and the coefficient of kinetic friction is $\mu k = 0.3$.

Solution

The body diagram in Figure 22 remains the same except that there is one additional force f_k in the negative *x* direction. Since $f_k = \mu_k N$, we need to find the normal force *N.* Considering the *y* direction we have (since $a_y = 0$) $N = mg = 19.6\ N$. The *x* equation is now $T - f_k = ma$ or $T - \mu k\ N = ma$. Substituting in the known values, we get

$$T - 0.3(19.6\ N) = (2.0\ kg)(10.0\ m/s^2) \qquad \text{or} \qquad T = 25.88\ N$$

Problem 32. A block of mass m = 5.0 kg slides from rest on a horizontal frictionless surface under the action of a force of 60 N in a direction 40 °above the positive *x* axis. How fast is the block moving at the end of 6 s?

Solution

The situation is depicted in Figure 23, where, instead of having a separate body diagram, all the forces acting on the block are directly drawn in. Only the *x* motion is of interest, and $F_x = ma_x$ yields

$$(60\ N)\ cos\ 40\ ° = (5.0\ kg)a \qquad \text{or} \qquad a = 9.2\ m^{-2}$$

Since we are starting from rest we have $V_x = at = (9.2\ \text{m/s}^2)(6\ \text{s}) = 55.2\ \text{m/s}$

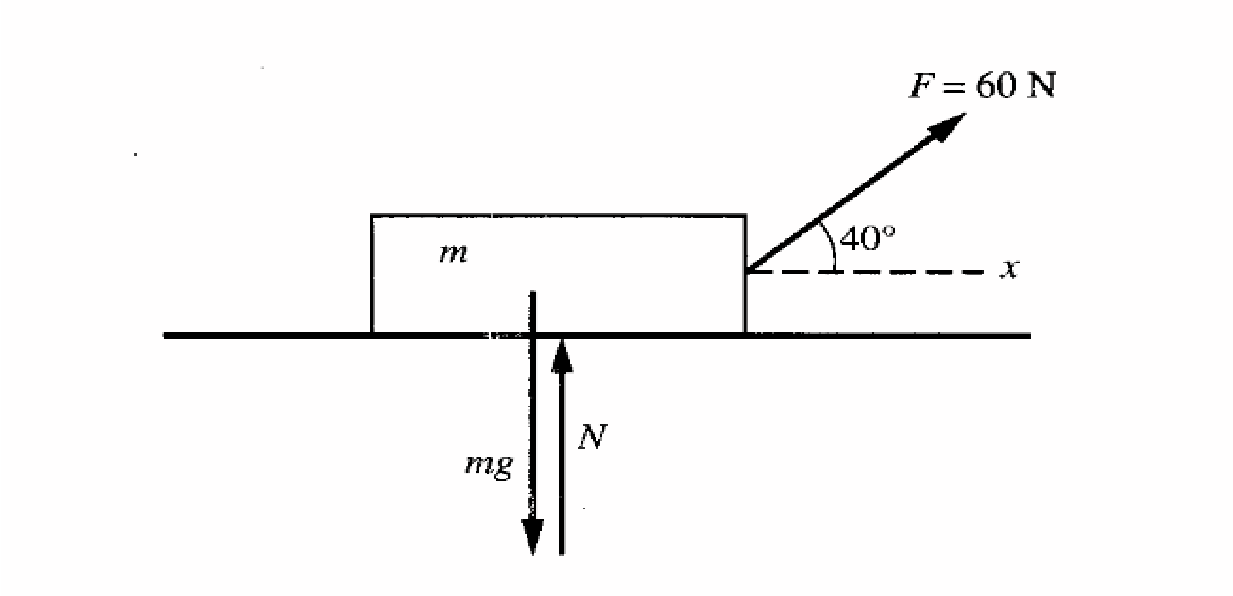


Figure 23

Problem 33. A block of mass $m = 12\ kg$ slides down a frictionless inclined plane of angle $50°$. What is the acceleration?

Solution

The situation is shown in Figure 24. Since we know the motion of the block will be down the incline, we choose our x axis down along the incline. Since there is no friction, the only force with a component along the incline is the weight $w = mg$. Then

$$mg\sin 50° = ma \text{ or } \quad a = g\sin 50° = (9.8\ ms^{-2})\ (0.766) = 7.51\ ms^{-2}$$

Note.: The acceleration is independent of the mass, just as for the case of freely falling objects. Indeed, if the angle of the incline is any angle *e*, the acceleration is $a = g\sin\theta$.

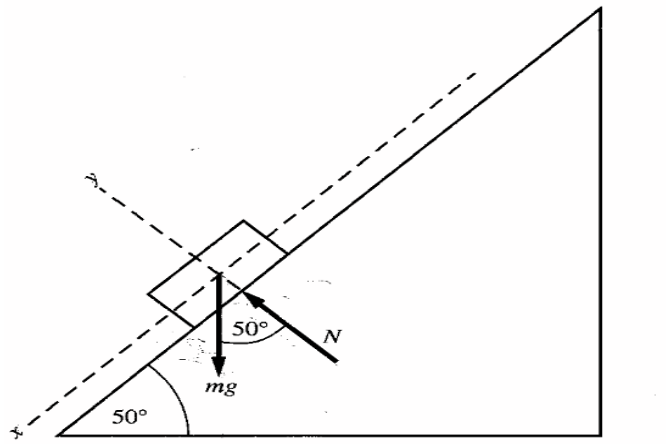


Figure 24

Problem 34. Suppose that in Problem 33 there is friction, with $\mu k = 0.2$. Find the acceleration.

Solution

A frictional force $f_k = \mu_k N$, acting up the incline (in the negative x direction), must be added to the forces already shown in Figure 24. Since we have equilibrium along the y *axis*,

$$N = mg\cos 50° \quad \text{and} \quad f_k = \mu_k mg\cos 50°$$

Then, for the x motion, $mg \sin 50° - \mu_k mg\ 50° = ma$. Dividing out the mass we obtain

$$a = g \sin 50° - \mu_k g \cos 50° = g(\sin 50° - \mu k \cos 50°)$$

$$= (9.8\ m/s^2)\ (0.766 - 0.2\ 0.643) = 6.25\ m/s^2$$

Problem 35. A child weighing 80 lb stands on a bathroom scale in an elevator. Find his "effective weight" as read on the scale, if the elevator is *(a)* moving downward at constant speed; *(b)* moving upward at constant speed; (c) accelerating upward at 8.0 ft/s^2; *(d)* accelerating upward at 32 ft/s^2; (e) accelerating downward at 8.0 ft/s^2; *(f)*accelerating downward at 32 ft/s^2.

Solution

The child is under the action of two forces, the weight $w = mg$ downward and the normal force N of the scale upward. (The bathroom scale reads the value of N, which is what we call the "effective weight".) We choose our positive direction upward.

(a), (b). In these two cases the acceleration is zero, so the child is in equilibrium, and we must have $N = mg$ = 80 lb, the true weight.

(c). Now $N - mg = ma$ or

$$N = m(g + a) = (w/g)(g + a) = w(g + a)/g = (80\ lb)(32 + 8)/32 = (80\ lb)\left(\frac{40}{32}\right)$$
$$= 100\ lb.$$

Note. This "effective weight" is not just a mathematical result. The child will actually feel heavier. Just as the scale pushes up with a force greater than the weight to give the entire child an upward acceleration, so too the lower half of the child must push up on the upper half with a greater than usual force to give that half its acceleration. Indeed, each part of the body must exert a proportionately greater force on every other part, hence the feeling of weighing more.

(d) We still have $N = w(g + a)/g$, but now a = 32 ft/s^2 Therefore N = (80 lb)(64/32)= 160 lb, or double the weight.

(e) Now $a=-8m/s^2$, and N = (80lb)m) = 60 lb.

(f) Now a = -32 ft/s^2 and $g + a = 0$, so $N = 0$.

Note. The answer to part *(f)* is not surprising because the child is accelerating downward with the acceleration of gravity, which is called "free fall." The child in fact feels weightless since no forces other than gravity can be acting on any given part of his body. Thus, the usual forces exerted by different parts of the body on each other are not there, and it feels strange. A satellite moving around the earth is also in free fall, which is why the astronauts inside feel weightless.