OSUN STATE UNIVERSITY
General Physics I

(PHY101)
UNIFORM CIRCULAR, SIMPLE HARMONIC AND ROTATIONAL MOTIONS

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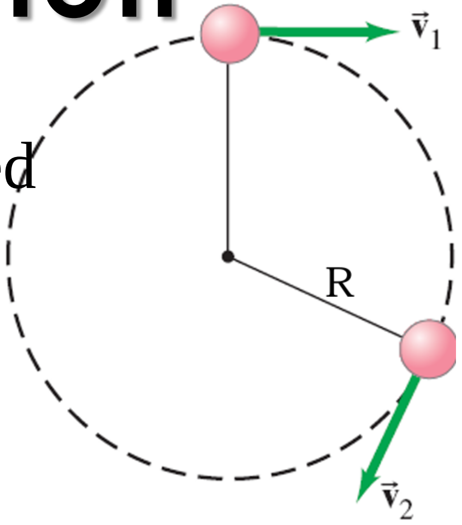
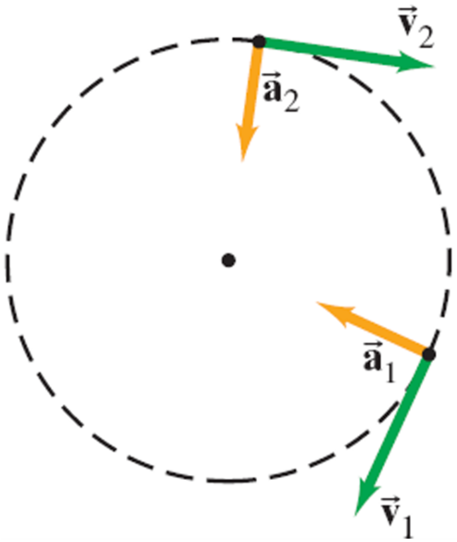
Uniform Circular Motion



- **Uniform circular motion**: motion in a circle at constant speed
- Instantaneous velocity is always tangent to the circle.
- The magnitude of the velocity is constant :

$$v_1 = v_1 = v$$

- **Centripetal acceleration**: The acceleration, called the centripetal acceleration, points toward the center of the circle.



- The magnitude of centripetal acceleration:

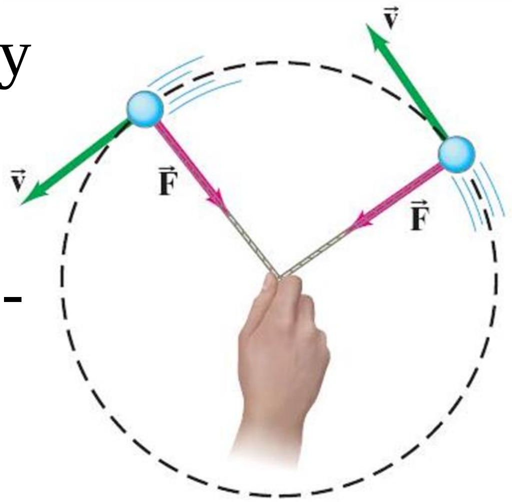
$$a_R = \frac{v^2}{R}$$

Dynamics of Uniform Circular Motion

- For an object to be in uniform circular motion, Newton's 2nd law requires a net force acting on it. This net force is called **centripetal force**:

$$\Sigma F_R = ma_R = m\frac{v^2}{r}.$$

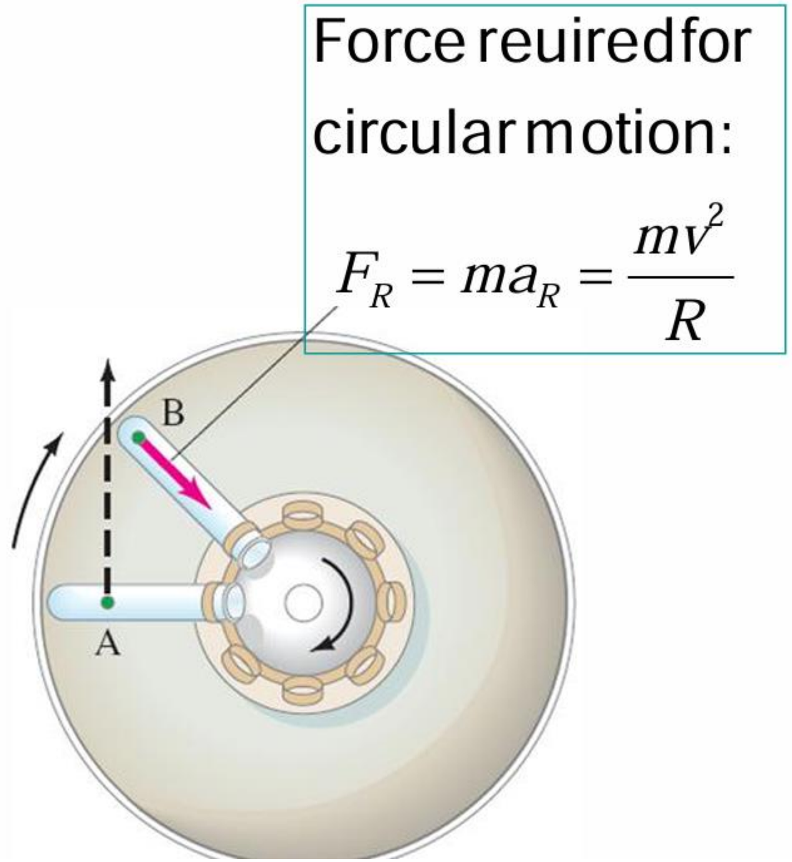
- Physically, the centripetal force can be the tension in a string, the gravity on a satellite, the normal force of a ring, etc.

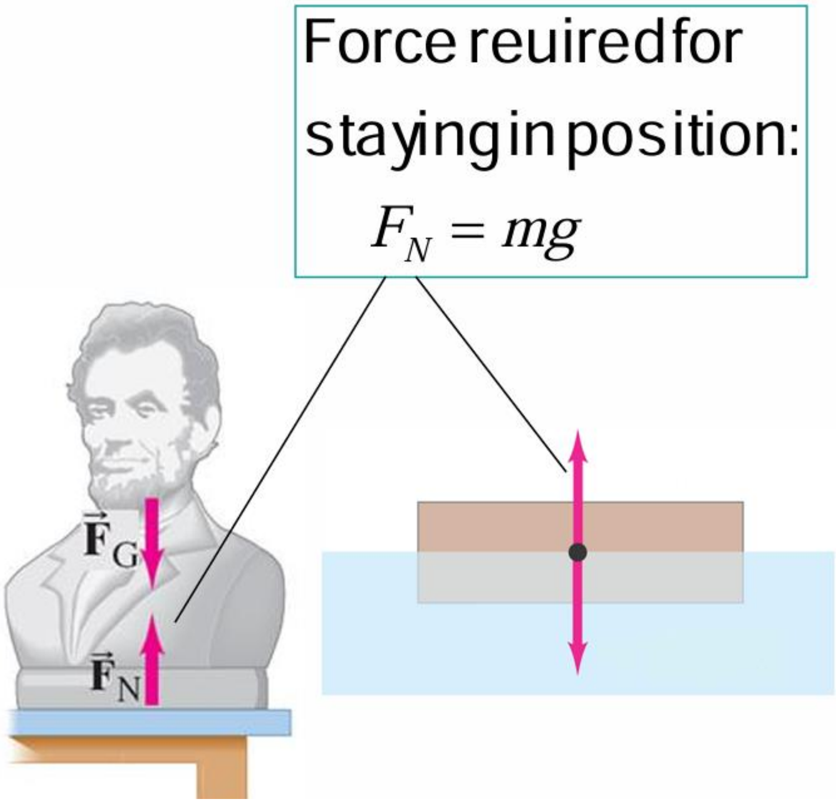


- Note: Don't count the centripetal force as an additional force in the free-body-diagram! It refers to the required net force for circular motion.

Centrifuge

A centrifuge works by spinning very fast. An object in the tube requires a large centripetal force. When the liquid can't provide such a large force, the object will move (sink) to the end of the tube.





Example: Ultracentrifuge.

The rotor of an ultracentrifuge rotates at 50,000 rpm (revolutions per minute). A particle at the top of a test tube is 6.00 cm from the rotation axis. Calculate its centripetal acceleration, in "*g*'s."

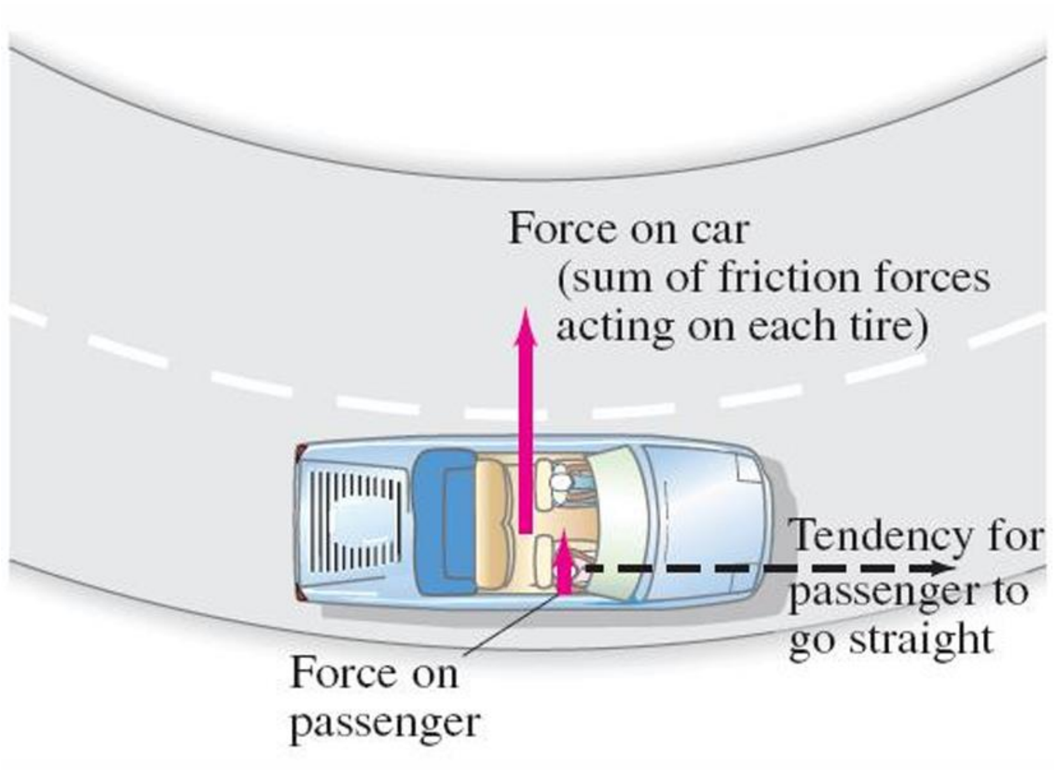
$$\text{Radius}: \quad R = 6.00\,cm = 0.0600\,m,$$

$$\text{Period}: \quad \mathrm{T} = \frac{1\,min}{50000\,rev} = \frac{60\,sec}{50000\,rev} = 1.2\times10^{-3}\ sec,$$

$$\text{Speed}: \quad v = \frac{2\pi R}{T} = \frac{2\pi\,(0.06\,)}{1.2\times10^{-3}} = 314\,m/s,$$

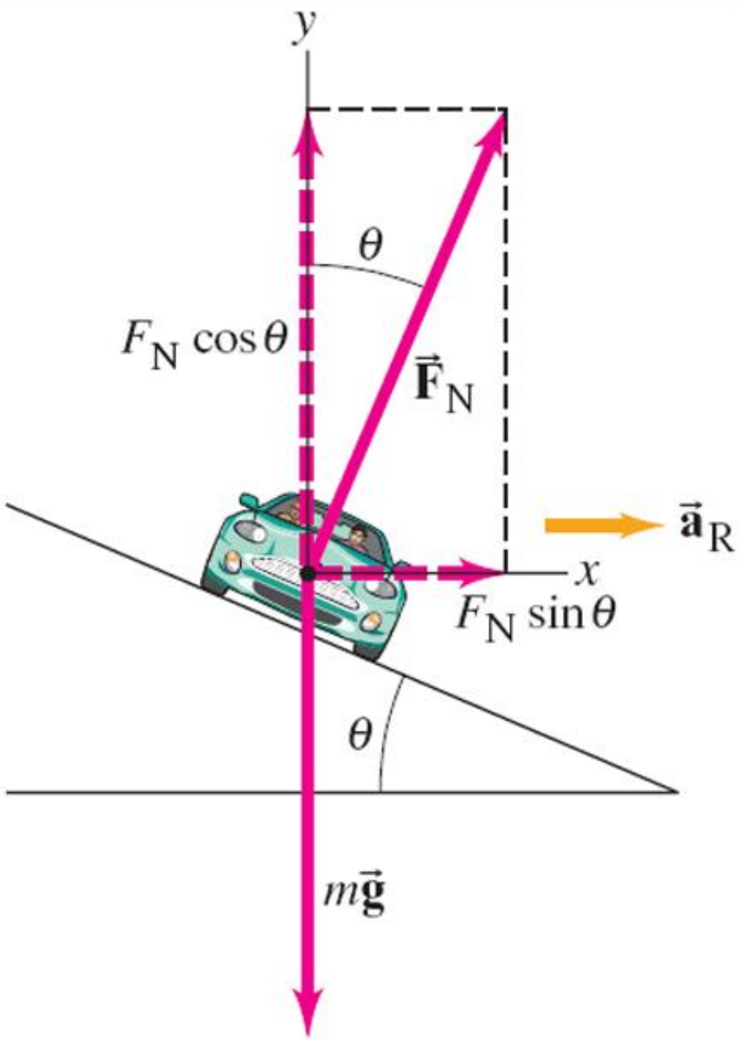
Highway Curves: Banked and Unbanked

- When a car goes around a curve, of which the curve is an arc. If there must be a net force toward the center of the circle. If the road is flat, that force is supplied by friction



- If the frictional force is insufficient, the car will tend to move more nearly in a straight line, as the skid marks show.

Highway Curves: Banked and Unbanked



Banking the curve can help keep cars from skidding. When the curve is banked, the centripetal force can be supplied by the horizontal component of the normal force. In fact, for every banked curve, there is one speed at which the entire centripetal force is supplied by the horizontal component of the normal force, and no friction is required.

Example: Banking angle

- (a) For a car traveling with speed v around a curve of radius r, determine a formula for the angle at which a road should be banked so that no friction is required. (b) What is this angle for an expressway off-ramp curve of radius 50 m at a design speed of 50 km/h?

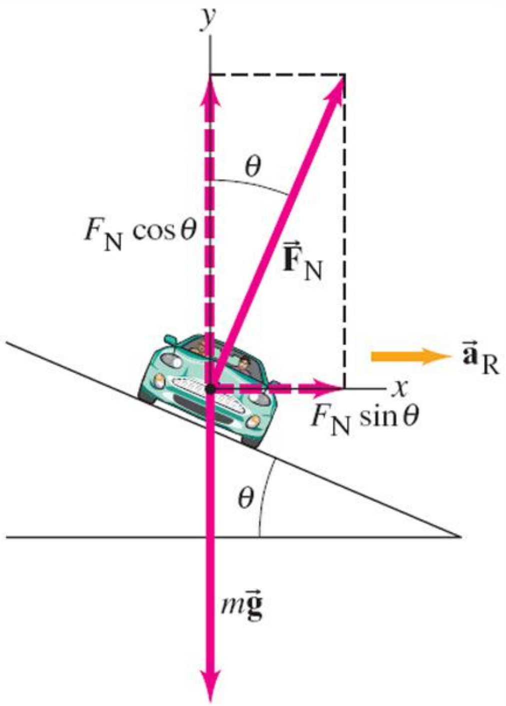
(a) $$F_N \sin\theta = m\frac{v^2}{R}$$

$$F_N \cos\theta - mg = 0$$

$$\tan\theta = \frac{v^2}{Rg}$$

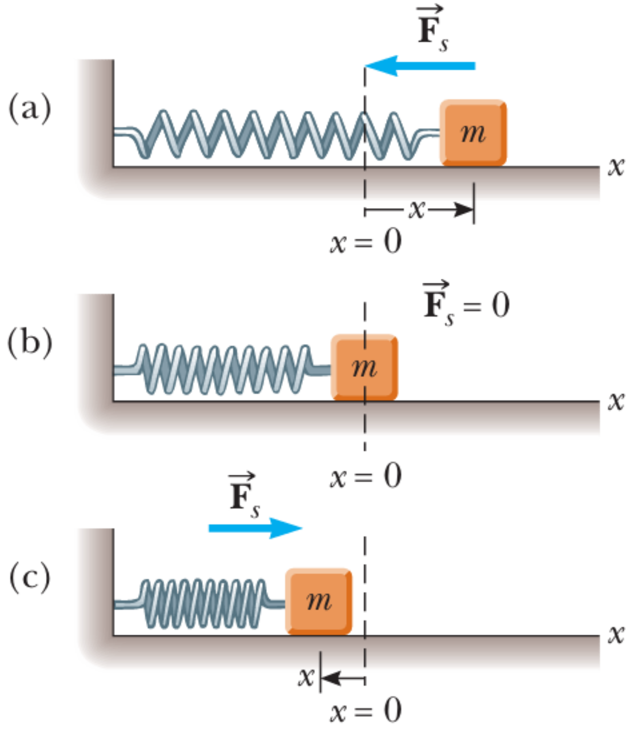
(b) $$R = 50\text{m},\quad v = 50\,km/h = 13.89\,m/s$$

$$\theta = \tan^{-1}\frac{v^2}{Rg} = \tan^{-1}\frac{13.89^2}{50g} = 22^\circ$$



Simple Harmonic Motion (SHM)

- If the spring is stretched or compressed a small distance x from its unstretched or equilibrium position and then released, it exerts a force F on the object
- Then, according to Hooke´s law, $F = -kx$
- The value of spring constant k is a measure of the stiffness of the spring. Stiff springs have large k values, and soft springs have small k values.
  - Because the spring force always acts toward the equilibrium position, it is sometimes called a restoring force.



Simple Harmonic Motion

- A restoring force always pushes or pulls the object toward the equilibrium position.
- Simple harmonic motion occurs
  - ✓ when the net force along the direction of motion obeys Hooke’s law.
  - ✓ when the net force is proportional to the displacement from the equilibrium point and is always directed toward the equilibrium point.
- Not all periodic motions over the same path can be classified as simple harmonic motion.

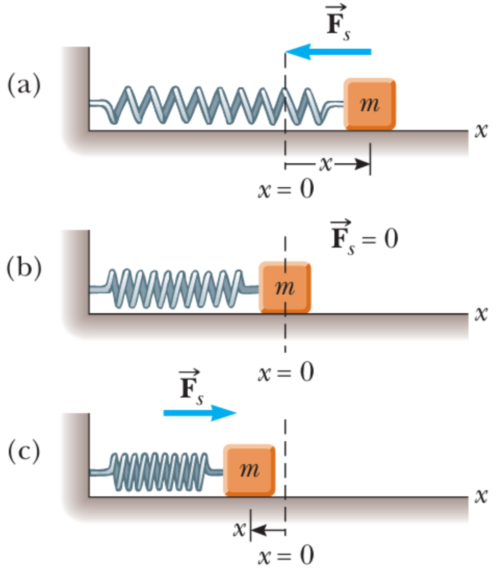
SHM: Three Main Concepts

The following three concepts are important in discussing any kind of periodic motion:

- The amplitude ***A***: is the maximum distance of the object from its equilibrium position. In the absence of friction, an object in SHM oscillates between the positions $\boldsymbol{x} = -\boldsymbol{A}$ and $\boldsymbol{x} = +\boldsymbol{A}$.
- The period ***T*** is the time it takes the object to move through one complete cycle of motion, from $\boldsymbol{x} = +\boldsymbol{A}$ to $\boldsymbol{x} = -\boldsymbol{A}$ and back to $\boldsymbol{x} = +\boldsymbol{A}$.
- The frequency ***f*** is the number of complete cycles or vibrations per unit of time, and is the reciprocal of the period ($\boldsymbol{f} = {}^{1}/_{\boldsymbol{T}}$)

SHM> Example 1

- A 0.350-kg object attached to a spring of force constant 1.30×10^2 N/m is free to move on a frictionless horizontal surface, as in the Figure. If the object is released from rest at x 0.100m, find the force on it and its acceleration at x 0.100m.



Solution:

$$F_s = -kx$$

$$F_{max} = -kA = -(1.30 \times 10^2 \text{ N/m})(0.100 \text{ m})$$

$$= -13.0 \text{ N}$$

$$ma = F_{max}$$

$$a = \frac{F_{max}}{m} = \frac{-13.0 \text{ N}}{0.350 \text{ kg}} = -37.1 \text{ m/s}^2$$

- **Exercise For the same spring and mass system, find the force exerted by the spring and the position x when the object's acceleration is 9.00 m/s².**

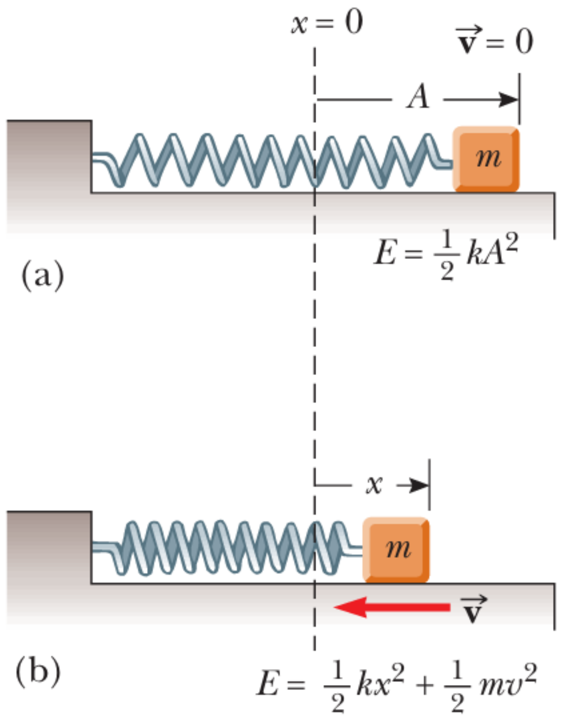
Answers: 3.15 N, 2.42 cm

SHM: Velocity as a function position

•From conservation of energy for object undergoing periodic motion as a function of position.

- The object in question is initially at its maximum extension A (Fig. a) and is then re leased from rest. The initial energy of the system is entirely elastic potential energy stored in the spring, $^1/_2\, kA^2$.
- As the object moves toward the origin to some new position x (Fig. b), the potential energy stored in the spring is reduced to the system is equal to: $^1/_2\, kx^2 + {^1/_2}\, mv^2$.
- Therefore: $^1/_2\, kA^2 = {^1/_2}\, kx^2 + {^1/_2}\, mv^2$

$$v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)}$$



SHM> Example 2

A 0.500-kg object connected to a light spring with a spring constant of 20.0 N/m oscillates on a frictionless horizontal surface. (a) Calculate the total energy of the system and the maximum speed of the object if the amplitude of the motion is 3.00 cm. (b) What is the velocity of the object when the displacement is 2.00 cm? (c) Compute the kinetic and potential energies of the system when the displacement is 2.00 cm

SHM> Example 1; Solution

(a)

$$E = KE + PE_g + PE_s$$
$$= 0 + 0 + \tfrac{1}{2}kA^2 = \tfrac{1}{2}(20.0\ \mathrm{N/m})(3.00 \times 10^{-2}\ \mathrm{m})^2$$
$$= 9.00 \times 10^{-3}\ \mathrm{J}$$

SHM> Example 2; Solution

a) Calculate the total energy of the system and the maximum speed of the object if the amplitude of the motion is 3.00 cm. (b) What is the velocity of the object when the displacement is 2.00 cm? (c) Compute the kinetic and potential energies of the system when the displacement is 2.00 cm

(a)

$$(KE + PE_g + PE_s)_i = (KE + PE_g + PE_s)_f$$

$$0 + 0 + \frac{1}{2}kA^2 = \frac{1}{2}mv_{\max}^2 + 0 + 0$$

$$\frac{1}{2}mv_{\max}^2 = 9.00 \times 10^{-3}\,\mathrm{J}$$

$$v_{\max} = \sqrt{\frac{18.0 \times 10^{-3}\,\mathrm{J}}{0.500\,\mathrm{kg}}} = 0.190\,\mathrm{m/s}$$

SHM> Example 2; Solution

(b) What is the velocity of the object when the displacement is 2.00 cm? (c) Compute the kinetic and potential energies of the system when the displacement is 2.00 cm

(b)

$$v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)}$$

$$= \pm\sqrt{\frac{20.0\ \mathrm{N/m}}{0.500\ \mathrm{kg}}((0.030\,0\ \mathrm{m})^2 - (0.020\,0\ \mathrm{m})^2)}$$

$$= \pm 0.141\ \mathrm{m/s}$$

(c) $KE = \frac{1}{2}mv^2 = \frac{1}{2}(0.500\ \mathrm{kg})(0.141\ \mathrm{m/s})^2 = 4.97 \times 10^{-3}\ \mathrm{J}$

$$PE_s = \frac{1}{2}kx^2 = \frac{1}{2}(20.0\ \mathrm{N/m})(2.00 \times 10^{-2}\ \mathrm{m})^2$$

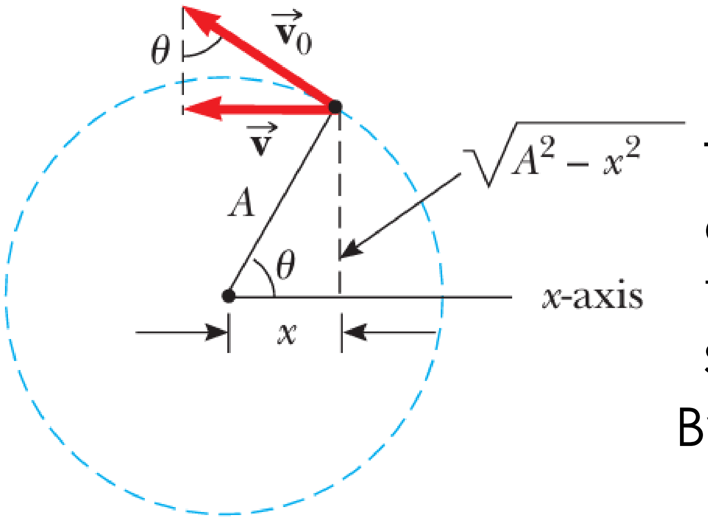
$$= 4.00 \times 10^{-3}\ \mathrm{J}$$

SHM> Circular Motion

We find that as the turntable rotates with constant angular speed, the shadow of the ball moves back and forth with simple harmonic motion.

Consider the an object that moves through the distance $2\pi A$ (the circumference of the circle) in the time **T**, the speed v_o of the ball around the circular path is

$$v_o = \frac{2\pi A}{T}$$



The motion of the shadow is equivalent to the horizontal motion of an object on the end of a spring. The radius A of the circular motion is the same as the amplitude A of the simple harmonic motion of the shadow.

By conservation of energy:

$$\tfrac{1}{2} kA^2 = \tfrac{1}{2} m{v_o}^2 \rightarrow \frac{A}{v_0} = \sqrt{\frac{m}{k}} \rightarrow T = 2\pi\sqrt{\frac{m}{k}} \rightarrow \omega = 2\pi f = \sqrt{\frac{k}{m}}$$

Quick Excercize:

An object of mass m is attached to a horizontal spring, stretched to a displacement A from equilibrium and released, undergoing harmonic oscillations on a friction less surface with period T_0. The experiment is then repeated with a mass of 4m. What's the new period of oscillation? (a) 2 T_0 (b) T_0(c) T_0 /2 (d) T_0 /4

Example 3: A 1.30 x10^3-kg car is constructed on a frame supported by four springs. Each spring has a spring constant of 2.00x10^4 N/m. If two people riding in the car have a combined mass of 1.60x10^2 kg, find the frequency of vibration of the car when it is driven over a pothole in the road. Find also the period and the angular frequency. Assume the weight is evenly distributed.

Assignment:

A 45.0-kg boy jumps on a 5.00-kg pogo stick with spring constant 3 650N/m. Find (a) the angular frequency, (b) the frequency, and (c) the period of the boy's motion. Answers (a) 8.54rad/s (b) 1.36Hz (c) 0.735s

SHM> Circular Motion: Example 3

Solution

$$m = \tfrac{1}{4}(m_{\text{car}} + m_{\text{pass}}) = \tfrac{1}{4}(1.30 \times 10^3\ \text{kg} + 1.60 \times 10^2\ \text{kg})$$

$$= 365\ \text{kg}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{2.00 \times 10^4\ \text{N/m}}{365\ \text{kg}}} = \boxed{1.18\ \text{Hz}}$$

$$T = \frac{1}{f} = \frac{1}{1.18\ \text{Hz}} = \boxed{0.847\ \text{s}}$$

$$\omega = 2\pi f = 2\pi(1.18\ \text{Hz}) = \boxed{7.41\ \text{rad/s}}$$

Rotational Motion

- In this context, we will find that an object remains in a state of uniform rotational motion unless acted on by a net torque. This principle is the equivalent of Newton's first law
- Further, the angular acceleration of an object is proportional to the net torque acting on it, which is the analog of Newton's second law. A net torque acting on an object causes a change in its rotational energy.
- Finally, torques applied to an object through a given time interval can change the object's angular momentum. In the absence of external torques, angular momentum is conserved

Torque

- Let $\vec{\mathbf{F}}$ be a force acting on an object, and let $\vec{\mathbf{r}}$ be a position vector from a chosen point O to the point of application of the force, with $\vec{F}$ perpendicular to $\vec{\mathbf{r}}$. The magnitude of the torque $\vec{\tau}$ exerted by the force is given by

$$\tau = rF \qquad (1)$$

where r is the length of the position vector and F is the magnitude of the force. SI unit: Newton-meter (Nm)

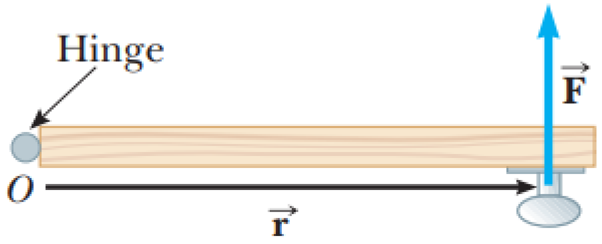


Figure 1. A bird's-eye view of a door hinged at O, with a force applied perpendicular to the door.

Torque: Moment of a Force

Note:

- ✓ When an applied force causes an object to rotate counterclockwise, the torque on the object is positive.
- ✓ When the force causes the object to rotate clockwise, the torque on the object is negative.
- ✓ When two or more torques act on an object at rest, the torques are added.
- ✓ If the net torque isn't zero, the object starts rotating at an ever-increasing rate.
- ✓ If the net torque is zero, the object's rate of rotation doesn't change.
- ❖ These considerations lead to the rotational analog of the first law: **the rate of rotation of an object doesn't change, unless the object is acted on by a net torque**

Example 1

- Two disgruntled businessmen are trying to use a revolving door, as in Figure 2. The door has a diameter of 2.60 m. The businessman on the left exerts a force of 625 N perpendicular to the door and 1.20 m from the hub's center, while the one on the right exerts a force of 8.50 x10^2 N perpendicular to the door and 0.800 m from the hub's center. Find the net torque on the revolving door.

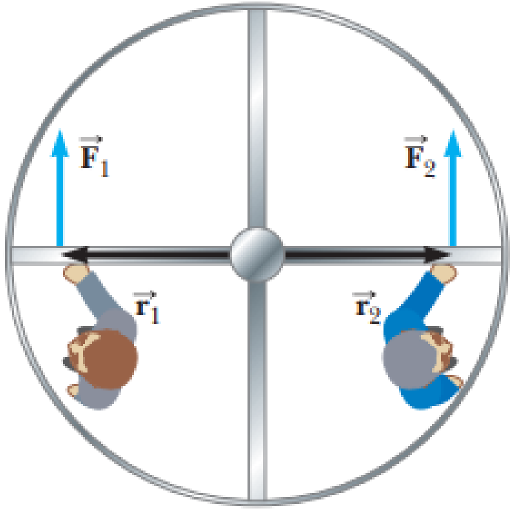


Figure 2.

Solution

Calculate the torque exerted by the first businessman. A negative sign must be supplied, because $\vec{\mathbf{F}}_1$, if unopposed, would cause a clockwise rotation: $\tau_1 = -r_1F_1 = -(1.2\text{ m})(625\text{ N}) =$ 7.50 x 10^2 N·m.

Calculate the torque exerted by the second businessman. The torque is positive because $\vec{\mathbf{F}}_2$, if unopposed, would cause a counterclockwise rotation. $\tau_2 = r_2F_2 = (0.800\ m)(8.50 \text{ x } 102\text{ N}) =$ **6.80 x 10^2 N·m.**

Sum the torques to find the net torque on the door: $\tau_{net} = \tau_1 + \tau_2 =$ **-7.0 x 10^1 N·m**

Vector products of Torque

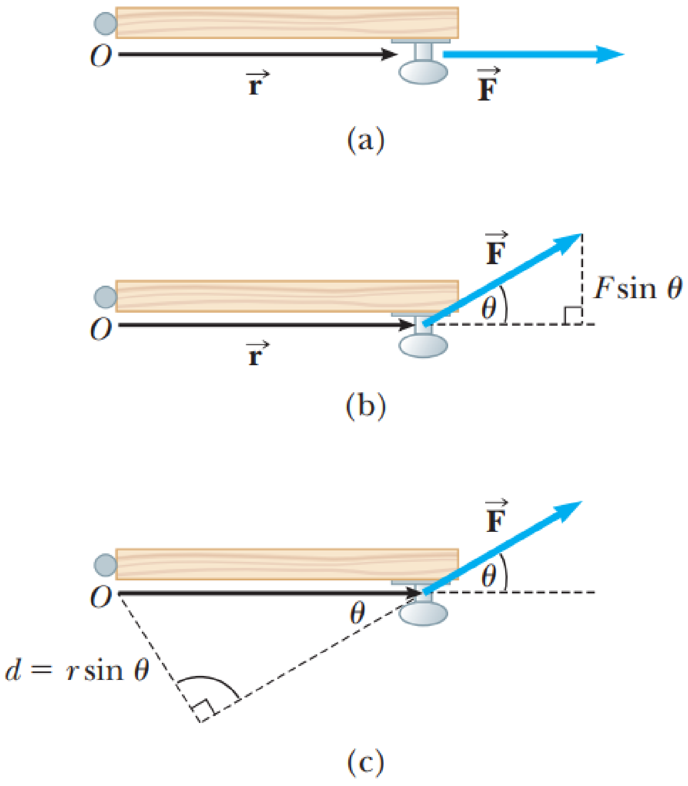


Figure 3 (a) A force acting at an angle θ = 180 exerts zero torque about the pivot O. (b) The part of the force perpendicular to the door, *F* sin θ , exerts torque *rF* sin θ about O. (c) An alternate interpretation of torque in terms of a lever arm d = *r* sin θ .

- **The applied force isn't always perpendicular to the position vector .**
- **This figure shows that the component of the force perpendicular to the door is F sin θ , where θ is the angle between the position vector $\vec{r}$ and the force $\vec{F}$. When the force is directed away from the axis, θ = 0, sin (0^o) = 0, and F sin (0^o) = 0. When the force is directed toward the axis, θ = 180^o and F sin (180^o) = 0. The maximum absolute value of F sin θ is attained only when $\vec{F}$ is perpendicular to $\vec{r}$ —that is, when θ = 90^o or θ = 270^o. These considerations motivate a more general definition of torque:**
- **Let $\vec{F}$ be a force acting on an object, and let $\vec{r}$ be a position vector from a chosen point O to the point of application of the force. The magnitude of the torque $\vec{\tau}$ exerted by the force is**
- $\tau = rF\sin\theta$ **(1)**

where r is the length of the position vector, F the magnitude of the force, and θ the angle between $\vec{r}$ and $\vec{F}$. SI unit: Newton-meter (N m)

Example 2

- (a) A man applies a force of F 3.00 102 N at an angle of 60.0 to the door of Figure 8.5a, 2.00 m from the hinges. Find the torque on the door, choosing the position of the hinges as the axis of rotation. (b) Suppose a wedge is placed 1.50 m from the hinges on the other side of the door. What minimum force must the wedge exert so that the force applied in part (a) won't open the door?

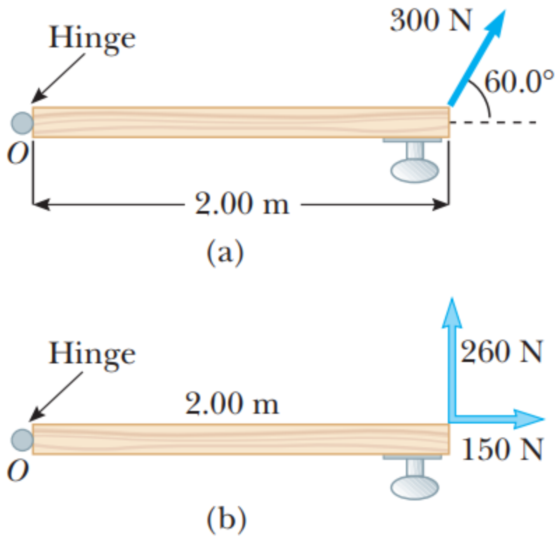


Figure 4. (a) Top view of a door being pushed by a 300-N force. (b) The components of the 300-N force (Example 2).

Solution

(a) Compute the torque due to the applied force exerted at 60.0°. Substitute into the general torque equation:

$$\tau_F = rF\sin\theta = (2.00\ \text{m})(3.00\times10^2\ \text{N})\sin 60.0^0 = (2.00\ \text{m})(2.60\times10^2\ \text{N}) = 5.20\times10^2\ \text{Nm}$$

(b) Calculate the force exerted by the wedge on the other side of the door. Set the sum of the torques equal to zero: $\tau_{hinge} + \tau_{wedge} + \tau_F = 0$ The hinge force provides no torque because it acts at the axis (r = 0). The wedge force acts at an angle of -90.0°, opposite F_y.

$$0 + F_{wedge}(1.50\ \text{m})\sin(-90.0^\circ) + 5.20\times10^2\ \text{N m} = 0$$

$$F_{wedge} = 347\ \text{N}$$

Exercise 1

A man ties one end of a strong rope 8.00 m long to the bumper of his truck, 0.500 m from the ground, and the other end to a vertical tree trunk at a height of 3.00 m. He uses the truck to create a tension of 8.00×10^2 N in the rope. Compute the magnitude of the torque on the tree due to the tension in the rope, with the base of the tree acting as the reference point. Answer 2.28×10^3 N m

TORQUE AND THE TWO CONDITIONS FOR EQUILIBRIUM

- An object in mechanical equilibrium must satisfy the following two conditions:
- 1. The net external force must be zero: $\sum \vec{F} = 0$
- 2. The net external torque must be zero: $\sum \vec{\tau} = 0$

- The first condition is a statement of translational equilibrium: The sum of all forces acting on the object must be zero, so the object has no translational acceleration, $\vec{a} = 0$. The second condition is a statement of rotational equilibrium: The sum of all torques on the object must be zero, so the object has no angular acceleration, $\vec{\alpha} = 0$. For an object to be in equilibrium, it must both translate and rotate at a constant rate.

- Because we can choose any location for calculating torques, it's usually best to select an axis that will make at least one torque equal to zero, just to simplify the net torque equation

Example 3

- **Problem A woman of mass m = 55.0 kg sits on the left end of a seesaw—a plank of length L = 4.00 m, pivoted in the middle as in Figure 5. (a) First compute the torques on the seesaw about an axis that passes through the pivot point. Where should a man of mass M = 75.0 kg sit if the system (seesaw plus man and woman) is to be balanced? (b) Find the normal force exerted by the pivot if the plank has a mass of m_{pl} = 12.0 kg. (c) Repeat part (b), but this time compute the torques about an axis through the left end of the plan**

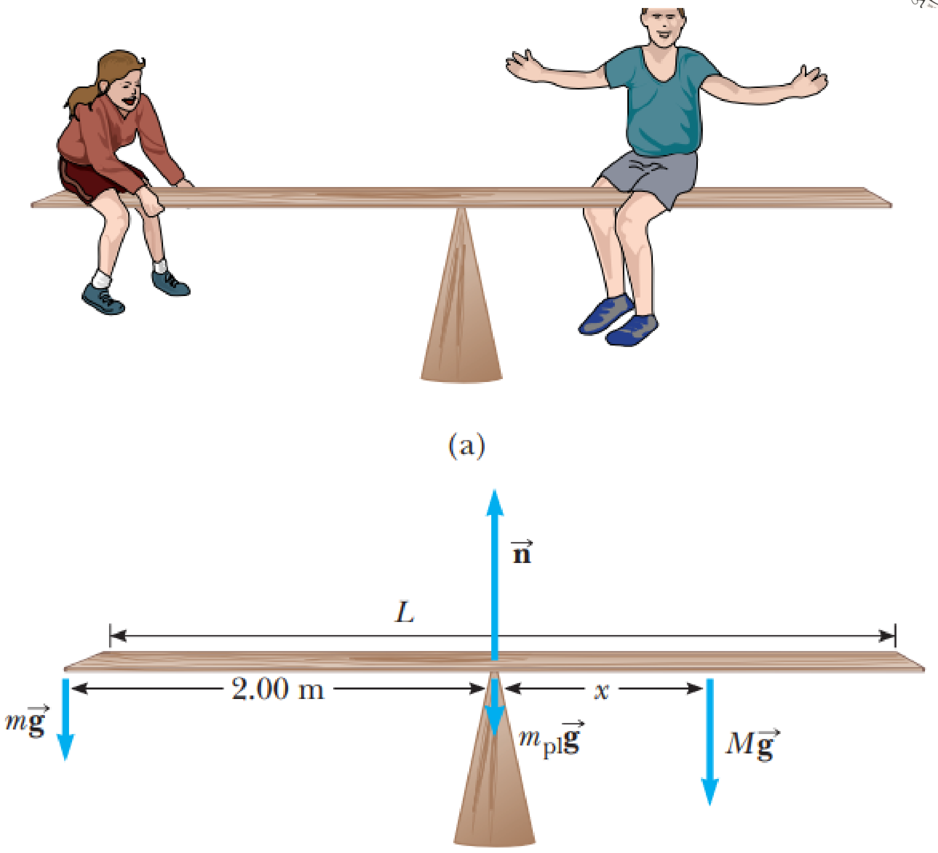


Figure 5 (a) (Example 2) Two people on a see-saw. (b) Free body diagram for the plank.

Solution: Example 3

- (a) Where should the man sit to balance the seesaw? Apply the second condition of equilibrium to the plank by setting the sum of the torques equal to zero:

$$\tau_{pivot} + \tau_{gravity} + \tau_{man} + \tau_{woman} = 0$$

- The first two torques are zero. Let x represent the man's distance from the pivot. The woman is at a distance L/2 from the pivot.

$$0 + 0 - Mgx + mg(L/2) = 0$$

- Solve this equation for x and evaluate it: $x = \frac{m(L/2)}{M} = \frac{(55.0\text{ kg})(2.00\text{ m})}{75.0\text{ kg}} = 1.47\text{ m}$

- (b) Find the normal force n exerted by the pivot on the seesaw. Apply for first condition of equilibrium to the plank, solving the resulting equation for the unknown normal force, *n*:

$$-Mg - mg - m_{pl}g + n = 0$$

$$n = (M + m + m_{pl})g$$

$$= (75.0\text{ kg} + 55.0\text{ kg} + 12.0\text{ kg})(9.80\text{ m/s}^2)$$

$$n = 1.39 \times 10^3\text{ N}$$

Solution: Example 3

- **(c) Repeat part (a), choosing a new axis through the left end of the plank. Compute the torques using this axis, and set their sum equal to zero. Now the pivot and gravity forces on the plank result in nonzero torques (Your answer should be: x = 1.46 m).**

Exercise 2

Exercise 2. Suppose a 30.0-kg child sits 1.50 m to the left of center on the same seesaw. A second child sits at the end on the opposite side, and the system is balanced. (a) Find the mass of the second child. (b) Find the normal force acting at the pivot point. Answers (a) 22.5 kg (b) 632 N

HAVE ANY

ENQUIRIES?

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