OSUN STATE UNIVERSITY, OSOGBO.
College of Science, Engineering and Technology.
Faculty of Basic and Applied Sciences
Department of Mathematical Sciences.
2021/2022 Harmattan Semester B.Sc. (Mathematics) Degree EXAMINATION

Course Code: MTH211/201 **Course Title:** Mathematical Method I.

Instruction(S): Answer FOUR Questions **Time Allowed:** 2 Hrs:30 min

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QUESTION ONE

(a) i. State the Mean Values Theorem, hence find the value of constant c for which the function $f(x) = 2x - x^2 - x^3$ Satisfy the mean value theorem on the interval $[-2,1]$

ii. Define homogenous function, hence prove that $s\frac{\partial \phi}{\partial s} + t\frac{\partial \phi}{\partial t} = n\phi$ and state all other deductions given that s *and* t are homogenous degree of order n.

(b) If $\phi = \sin^{-1}[x^3 + y^3]^{\frac{2}{5}}$ prove that $x^2\frac{\partial^2\phi}{\partial x^2} + 2xy\frac{\partial^2\phi}{\partial x\partial y} + y^2\frac{\partial^2\phi}{\partial y^2} = \frac{6}{5}\tan\phi\left[\frac{6}{5}\sec^2\phi - 1\right]$

(c) Given a function $\tan^{-1}(x) = \int \frac{1}{1+x^2}dx$ which is continuous on the interval [-1, 1], show that $\tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} + \cdots$. Prove further that $\pi = 4\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)}$

QUESTION TWO

(a) if $u = f(ax^2 + 2hxy + by^2)$ and $v = \phi(ax^2 + 2hxy + by^2)$ show that $\frac{\partial}{\partial y}\left(u\frac{\partial v}{\partial x}\right) = \frac{\partial}{\partial x}\left(u\frac{\partial v}{\partial y}\right)$

(b) If $z = 2(x+y) + \frac{y}{x}$ Prove that $x\left(\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x\partial y}\right) = y\left(\frac{\partial^2 z}{\partial y^2} - \frac{\partial^2 z}{\partial x\partial y}\right)$

(c) Suppose $u = f(x,y)$, where $x = e^r\cos\theta$ and $y = e^r\sin\theta$ prove that $e^{2r}\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 y}{\partial x^2}\right] = \frac{\partial^2 u}{\partial r^2} + \frac{\partial^2 y}{\partial\theta^2}$

QUESTION THREE

(a) i. Suppose $z = 0$ in the relation $z = f(x,y)$, deduce that $\frac{dy}{dx} = -\frac{p}{q}$. Prove further that $\frac{d^2y}{dx^2} = \frac{-q^2r - 2pqs + p^2t}{q^3}$

Where p,q,r,s,t represent their respective notations. (ii) Find $\frac{dy}{dx}$ If $(\cos x)^y = (\sin y)^x$.

(b) If $x = f(u,v)$ *and* $y = \phi(u,v)$ show that If $\frac{\partial u}{\partial x} = \frac{\varphi_v}{f_u\varphi_v - f_v f_v}$ and $\frac{\partial v}{\partial x} = \frac{-\varphi_u}{f_u\varphi_v - f_v\varphi_u}$ hence, find $\frac{\partial u}{\partial x}$ *and* $\frac{\partial v}{\partial x}$

Given that $x = u^2 - v^2$ *and* $y = uv$

(c) if $x^x y^y z^z = c$, show that at $x = y = z$, $\frac{\partial^2 z}{\partial x\partial y} = -(x\log xc)^{-1}$.

QUESTION FOUR

(a)Examine the following functions for extremum values

(i) $f(x,y)=y^2+4xy+3x^2+x^3$ (ii) $f(x,y)=x^3y^2(1-x-y)$ (iii) $xye^{-(2x+3y)}$

(b) If $f(x,y)$ and $\phi(x,y)$ are homogenous functions of x,y of degree p and q respectively, and $u=f(x,y)+\varphi(x,y)$ prove that $f(x,y)=\frac{1}{p(p-q)}\left\{x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}\right\}-\frac{q-1}{p(p-q)}\left\{x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial x}\right\}$

(c) Show that the maximum value of the function $\sum_{i=1}^{n}a_i y_i$ with constraint $\sum_{i=1}^{n}y_i^2=1$ is $\left[\sum a_i^2\right]^{\frac{1}{2}}$

QUESTION FIVE

(a) Let $f(x,y,z)$ be a function of three variables, connected with constraint $\varphi(x,y,z)=0$.
Prove that the Lagrange equation hold well for $f_x+\lambda\varphi_x=0,\ f_y+\lambda\varphi_y=0,\ f_z+\lambda\varphi_z=0$.

(b)Define the Jacobian of a function u,v,w with respect to x,y,z hence, if $u=xyz\ v=x^2+y^2+z^2,\ w=x+y+z$. Find $J\left[\frac{\partial(x,y,z)}{\partial(u,v,w)}\right]$ hence find the functional relationship between them.

(c)Evaluate the double integral i. $\int_0^2\int_0^{x^2} e^{-\frac{y}{x}}dydx$ ii. $\int_0^2\int_0^{x^2}\frac{xdxdy}{\sqrt{x^2+y^2}}dydx$

QUESTION SIX

(a) if $x^3+y^3-3axy=0$ find $\frac{dy}{dx},\frac{d^2y}{dx^2}$ and check the function for its extremum values

(b) If $(\lambda-x)^3+(\lambda-y)^3+(\lambda-z)^3=0$ find $J\left[\frac{\partial(u,v,w)}{\partial(x,y,z)}\right]$

(c) Duplicating the integral $I=\int_0^\infty e^{-x^2}dx$ and applying transformation $x=r\cos\theta, y=r\sin\theta$.

Show that $I=\frac{\sqrt{\pi}}{2}$

OSUN STATE UNIVERSITY, OSOGBO, NIGERIA

College of Science, Engineering and Technology

DEPARTMENT OF STATISTICS

2021/2022 Harmattan Semester Examination.

Course Title: Statistics for Physical Sciences and Engineering. Unit: 3 Units

Course Code: STA 221 Time: 2 Hrs 30 Minutes

Instruction: Answer any four questions.

Question One

a. Itemize the steps needed to solve ANOVA.

b. An experiment was run to determine whether four specific firing temperatures affect the density of a certain type of brick. The experiment led to the following data.

Temp. (^0F)	Density						
100	21.8	21.9	21.7	21.6	21.7	21.5	21.8
125	21.7	21.4	21.5	21.5	-	-	-
150	21.9	21.8	21.8	21.6	21.5	-	-
175	21.9	21.7	21.8	21.7	21.6	21.8	-

Does the firing temperature affect the density of the brick? Use $\alpha = 0.05$ (17.5 marks)

Question Two

The table below gives the Statistics marks, it represents the frequency distribution of 1,000 Students of College of Education, Osun State University.

Marks	0-9	10-19	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99
Frequency	20	41	98	149	192	197	152	97	44	10

Using the information above, draw (a) i. Bar chart ii. Histogram and iii. Pie chart.

(b) Compute Kurtosis and Skewness.

Question Three

Given $\beta_i = (X'X)^{-1}X'Y$ as the estimated parameter of a regression model,

$$X = \begin{pmatrix} 1 & 8 & 1 \\ 1 & 4 & 2 \\ 1 & 3 & 3 \\ 1 & 1 & 4 \\ 1 & 9 & 5 \end{pmatrix} \quad \text{and} \quad Y = \begin{pmatrix} 16 \\ 16 \\ 17 \\ 18 \\ 17 \end{pmatrix} \quad \text{Find } \beta_1, \beta_2 \text{ and } \beta_3$$

Question Four

A. State the properties of the Binomial distribution.

B. Investigation shows that out of every 8 patients treated with a new malaria vaccine, 6 patients are cured. If 5 patients are treated with the vaccine, what is the probability that

i. exactly one (1) patient is cured ii. more than three (3) patients are cured.

ii. at least three (3) patients are cured iv. Between two (2) and four (4) patients are cured.

C. Given that X is a random variable with probability density function

$$f(x)=\begin{cases}\frac{5}{6}(x^2-3tx) & ;1<x<3\\ 0 & ;elsewhere\end{cases},$$

Find i. the constant t ii. variance of x .

Question Five

a. The management of a large national chain of motels decided to estimate the mean cost per room of repairing damages made by its customers during a Bank holiday weekend. A random sample of 150 vacated rooms was inspected by the management. Its analysis estimated the mean repair cost to be ₦850.00 and the sample standard deviation to be ₦240. Construct a 95% confidence interval for the mean repair cost of all its rooms.

b. A manufacturer of electronic calculators is interested in estimating the fraction of defective units produced. A random sample of 800 calculators contains 10 defectives. Compute a 99% confidence interval on the fraction defective.

c. A rivet is to be inserted into a hole. A random sample of n =15 parts is selected and the hole diameter is measured. The sample standard deviation of the hole diameter measurement is s = 0.008 millimetres. Construct a 99% confidence interval for σ^2

Question Six

a. A bearing used in an automotive application is supposed to have a nominal inside diameter of 1.5 inches. A random sample of 25 bearings is selected and the average inside diameter of these bearings is 1.4975 inches. Bearing diameter is known to be normally distributed with standard deviation $\sigma = 0.01$ inch.

b. Test the hypothesis that the inside diameter is different from 1.5 using $\sigma = 0.01$. The life in hours of a battery is known to be approximately normally distributed, with standard deviation $\sigma = 1.25$ hours. A random sample of 10 batteries has a mean life of $\bar{x}$ 40.5 hours. Is there evidence to support the claim that battery life exceeds 40 hours? Use $\sigma =$ 0.05.

OSUN STATE UNIVERSITY, OSOGBO, NIGERIA
College of Science, Engineering and Technology
DEPARTMENT OF STATISTICS

2021/2022 Harmattan Semester Test.

Course Title: Statistics for Physical Sciences and Engineering. **Unit:- 3 Units**

Course Code: STA221 **Time: One Hour**

Instruction: Answer all questions.

SECTION A.

1. Differentiate between a point and interval estimate.

Use the information below to answer questions 2 and 3

Let two events A and B be defined on the same sample space. Suppose $p(B) = 0.35$ *and* $p(A \cup B) = 0.5$. Find $p(A)$; Such that;

2. Find $p(A)$; Such that; A and B are independent
3. Find $p(A)$; Such that; A and B are mutually exclusive.
4. A coin is loaded in such a way that a tail is thrice as likely to appear as a head, if the coin is tossed five times, find the probability that exactly two tails appear
5. In the simple linear regression model, the least squares estimator is derived by
6. If the Spearman's rank correlation coefficient of a set of four paired data is 0.8, find the sum of the squares of the difference between the ranks of each pair.
7. The line of best fit is always drawn on
8. Given that X is a random variable with probability density function

$$f(x) = \begin{cases} kx^2 & ; \; 1 < X < 3 \\ 0 & ; \; elsewhere \end{cases}$$

Find the constant k.

9. What is the difference between mean deviation and Standard deviation.
10. What is the condition for a distribution to be symmetric?

SECTION B

A. The table below gives the Statistics marks, representing the frequency distribution of 1,000

B. Students of the College of Education, Osun State University.

Marks	0-9	10-19	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99
Frequency	20	41	98	149	192	197	152	97	44	10

Use the Table above to compute kurtosis and skewness.

B. The average monthly consumption for a sample of 100 families is 1250 units. Assuming the standard deviation of electric consumption of all families is 150 units, construct a 95 percent confidence interval estimate of the actual mean electric consumption

A police department conducts a test on the brakes of 100 randomly selected vehicles on the highway. They found that 38 of them are in need of repairs.

i. Find a 99% confidence interval for the proportion of all vehicles driving with good brakes.

ii. Find a 99% confidence interval for the proportion of all vehicles driving with faulty brakes.