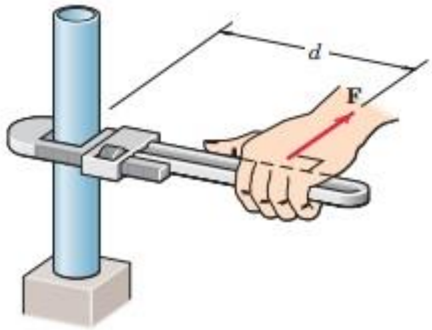
MOMENT AND COUPLE

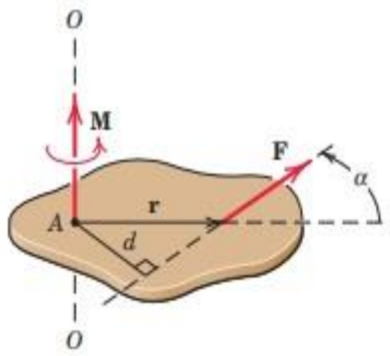
Moment of a Force: Scalar Formulation

In addition to the tendency to move a body in the direction of its application, a force can also tend to rotate a body about an axis. The axis may be any line which neither intersects nor is parallel to the line of action of the force. This rotational tendency is known as the moment M of the force. Moment is also referred to *as* torque.

Consider the pipe wrench shown in the figure below. One effect of the force applied perpendicular to the handle of the wrench is the tendency to rotate the pipe about its vertical axis. The magnitude of this tendency depends on both the magnitude F of the force and the effective length *d* of the wrench handle. Common experience shows that a pull which is not perpendicular to the wrench handle is less effective than the right-angle pull shown.



The figure below shows a two-dimensional body acted on by a force F in its plane. The magnitude of the moment or tendency of the force to rotate the body about the axis *O*-*O* perpendicular to the plane of the body is proportional both to the magnitude of the force and to the moment arm 'd', which is the perpendicular distance from the axis to the line of action of the force.



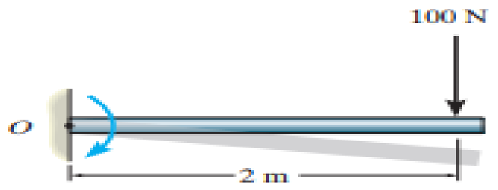
Therefore, the magnitude of the moment is defined as

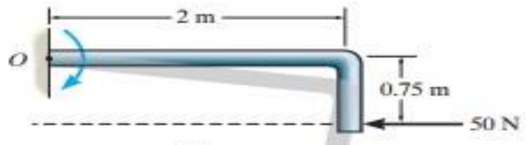
$M = F.d$

The moment is a vector M perpendicular to the plane of the body. The units of moment are that of force multiplied by length, for example N.m, lb.ft and so on. The sense of M depends on the direction in which F tends to rotate the body which may be represented as either clockwise or anticlockwise. As a convention in this course we will take the clockwise rotation as negative while the anticlockwise rotation will be taken as positive.

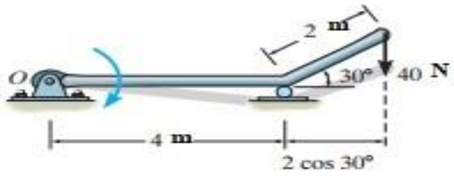
Example

For each of the cases illustrated below, determine the moment about point 'O'.

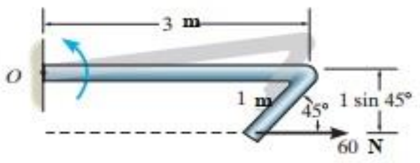




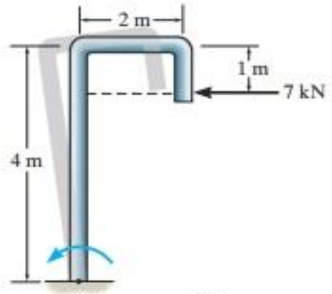
(b)



(c)

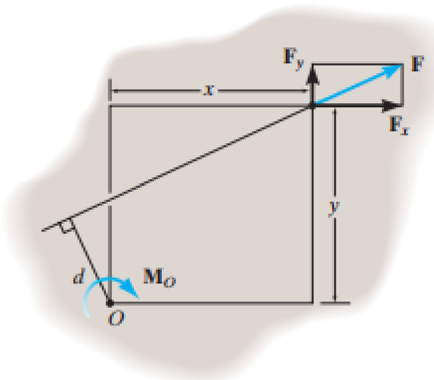


(d)



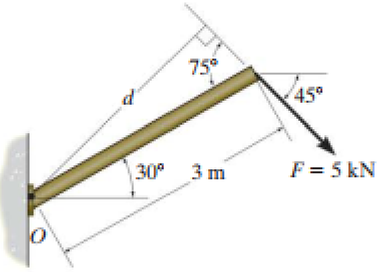
(e)

Principle of Moment (Varignon's Theorem)
Varignon's theorem says that the moment of a force about a point is equal to the sum of the moments of the components of the force about the same axis.
For two-dimensional problems, as shown below, we can use the principle of moments by resolving the force into its rectangular components and then determine the moment using a scalar analysis.

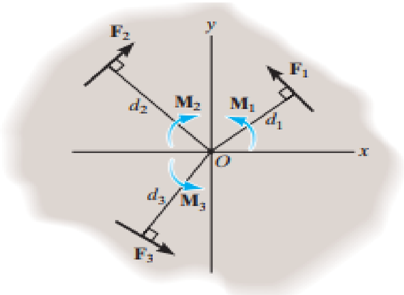


Therefore
$M_o = F_x y + F_y x$
This method is generally easier than finding the same moment using
$M_O = F.d$

Example
Determine the moment of the force below about the point O

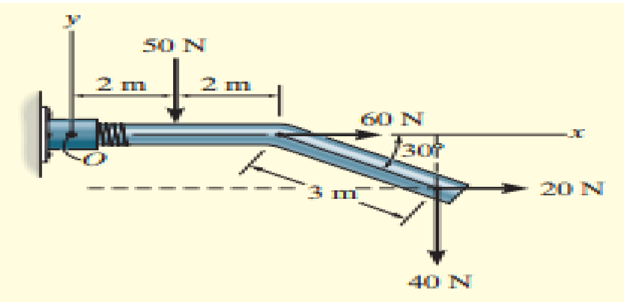


Resultant Moment of Several Coplanar Forces: For two-dimensional problems, where all the forces lie within the x–y plane, as shown in the figure below, the resultant moment $(M_R)_O$ about point O (the z axis) can be determined by finding the algebraic sum of the moments caused by all the forces in the system.

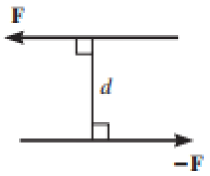


The resultant moment of the system above is therefore
$(M_R)_o = F_1 d_1 - F_2 d_2 + F_3 d_3$

Example
Determine the resultant moment of the four forces acting about point O

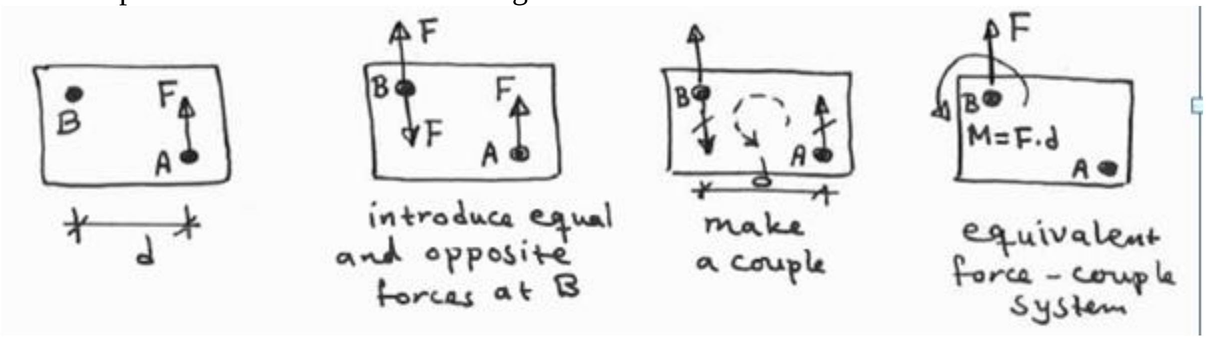


Moment of a Couple
A couple is defined as two parallel forces that have the same magnitude, but opposite directions, and are separated by a perpendicular distance d, as shown below. Since the resultant force is zero, the only effect of a couple is to produce an actual rotation, or if no movement is possible, there is a tendency of rotation in a specified direction.

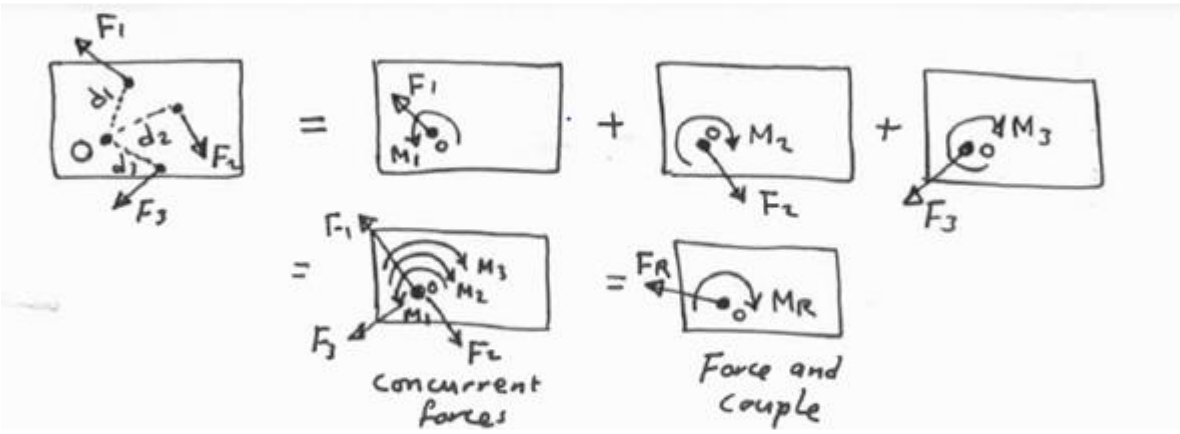


Uses of Couple in Statics

1. It is used for changing the line of action of the force, for example in moving the force F from point A to B as shown in the figure below

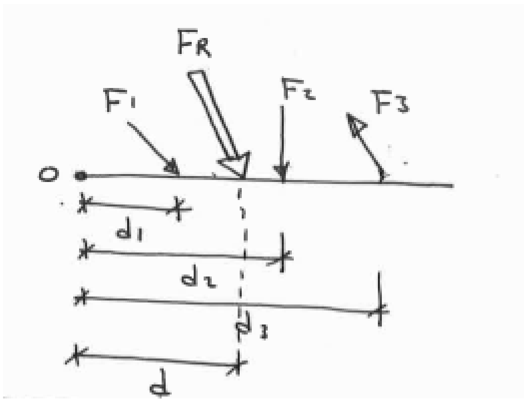


2. It is used in reducing a system of forces to a force and couple



Using the Concept of moment to find the resultant of Non-concurrent Forces

In finding the magnitude, direction and location of the resultant force F_R as shown below.



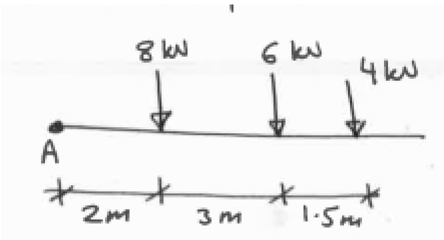
The magnitude $F_R = \sqrt{(F_{RX})^2 + (F_{RY})^2}$

The direction $\theta = \tan^{-1}\frac{F_{RY}}{F_{Rx}}$

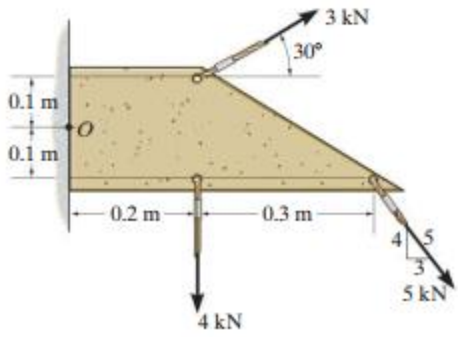
The location $(F_R)_y.d = (F_1)_y.d_1 + (F_2)_y.d_2 - (F_3)_y.d_3$
That is $(M_R)_O = \sum M_O$ - Varignon's Theorem.
This equation gives the location of'd'

Example

1. Determine the magnitude, direction and location of the resultant force from point 'A'

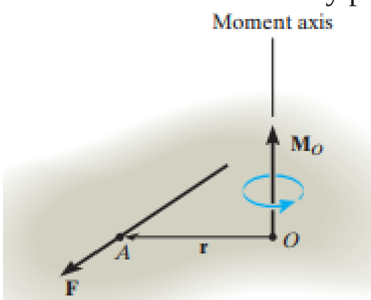


2. Replace the force and couple system shown in the figure below by an equivalent resultant force and couple moment acting at point O



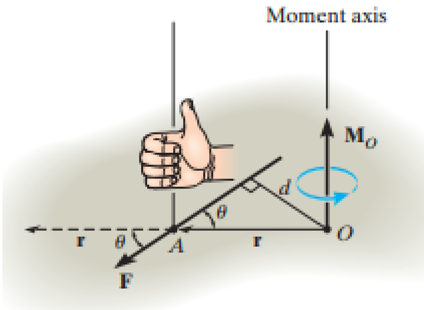
Moment of a Force: Vector Formulation

The moment of a force F about point O, or actually about the moment axis passing through O and perpendicular to the plane containing O and F, as shown in the figure below, can be expressed using the vector cross product as
$M_o = r \times F$
where r represents a position vector directed from O to any point on the line of action of F.



The magnitude of the cross product is defined as
$M_o = rF \sin\theta$

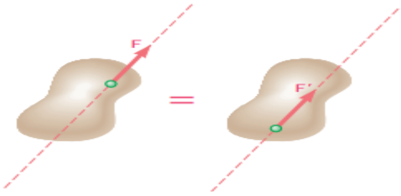
where the angle θ is measured between the tails of r and F. To establish this angle, r must be treated as a sliding vector so that θ can be constructed properly, as shown below.



Since the moment arm $d = r\sin\theta$ then
$M_o = rF\sin\theta = F(r\sin\theta) = Fd$

Principle of Transmissibility

The principle of transmissibility states that the conditions of equilibrium or motion of a rigid body will remain unchanged if a force F acting at a given point of the rigid body is replaced by a force F' of the same magnitude and same direction, but acting at a different point, provided that the two forces have the same line of action as shown in the figure below.



Cartesian Vector Formulation

If we establish *x*, *y*, *z* coordinate axes, then the position vector r and force F can be expressed as Cartesian vectors as

$$M_o = r \times F = \begin{vmatrix} i & j & k \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Where r_x, r_y, r_z represent the x, y, z components of the position vector drawn from point O to any point on the line of action of the force Fx, Fy, Fz represent the x, y, z components of the force vector.
If the determinant of the equation above is expanded, we have

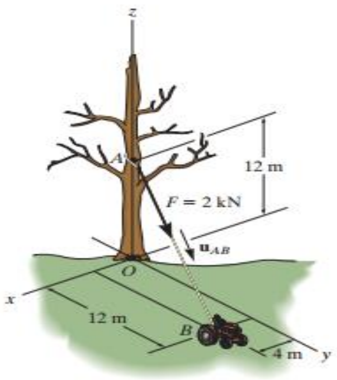
$$M_o = (r_yF_z - r_zF_y)i - (r_xF_z - r_zF_x)j + (r_xF_y - r_yF_x)k$$

Resultant Moment of a System of Forces

If a body is acted upon by a system of forces, the resultant moment of the forces about point *O* can be determined by vector addition of the moment of each force. This resultant can be written symbolically as
$(M_R)_o = \sum(r \times F)$

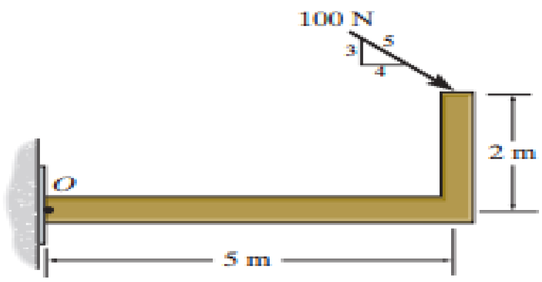
Example

1. Determine the moment produced by the force F in the figure below about point O. Express the result as a Cartesian vector.

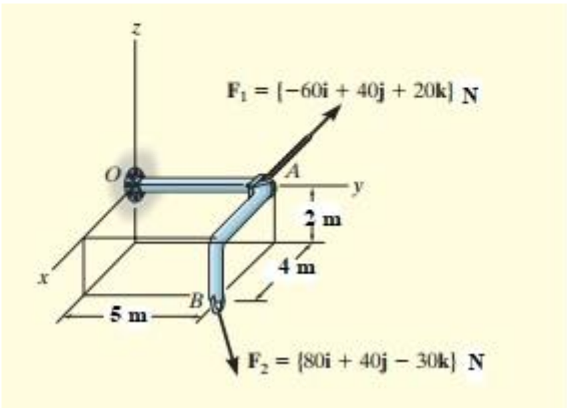


Practice Questions

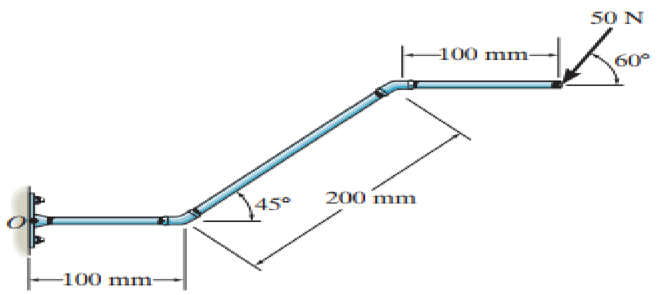
1. Determine the moment of the force about point O



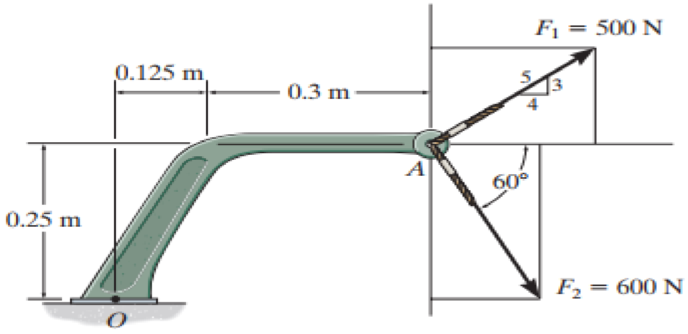
2. Two forces act on the rod shown in the figure below. Determine the resultant moment they create about the flange at *O*. Express the result as a Cartesian vector.



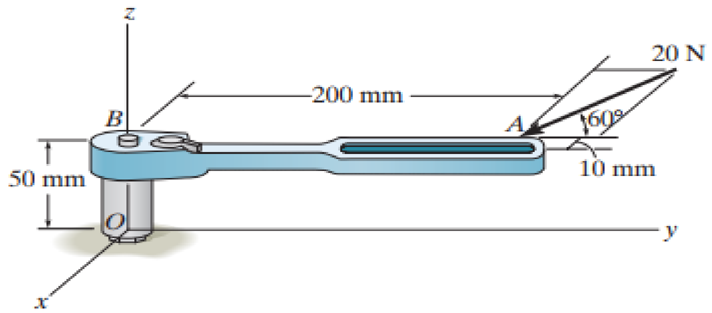
3. Determine the moment of the force about point *O*. Neglect the thickness of the member



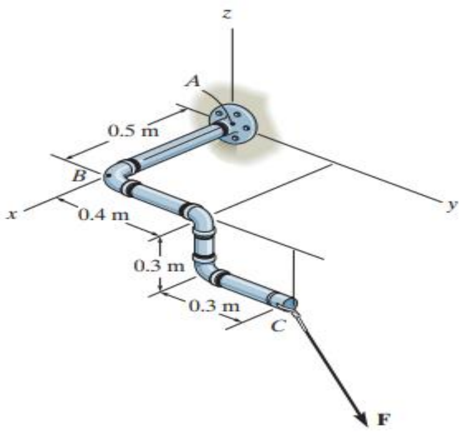
4. Determine the resultant moment produced by the forces about point O



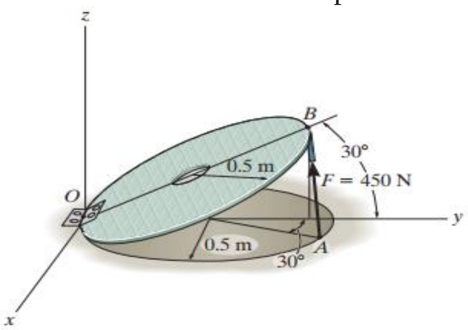
5. The 20-N horizontal force acts on the handle of the socket wrench. What is the moment of this force about point *B*. Specify the coordinate direction angles α, β, γ of the moment axis.



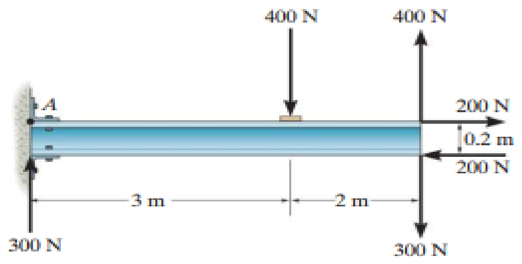
6. The pipe assembly is subjected to the force of **F** = {600**i** + 800**j** - 500**k**} N. Determine the moment of this force about point *B*



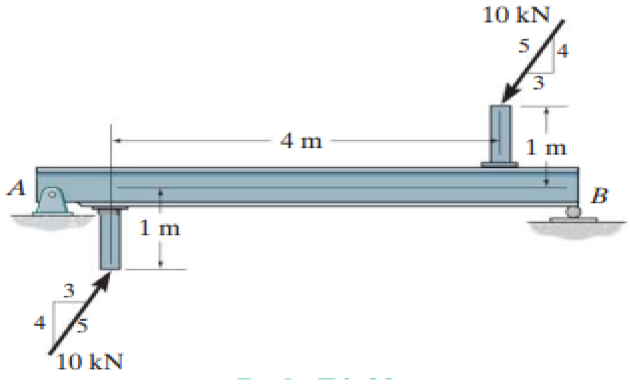
7. Strut *AB* of the 1-m-diameter hatch door exerts a force of 450 N on point *B*. Determine the moment of this force about point *O*.



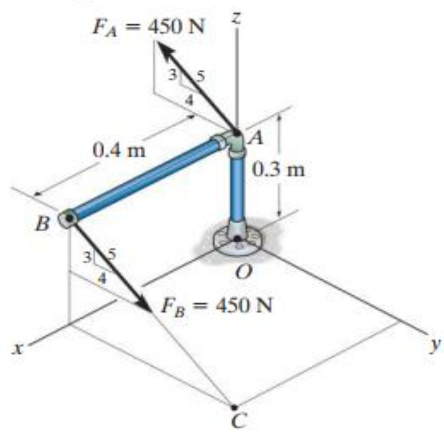
8. Determine the resultant couple moment acting on the beam shown below



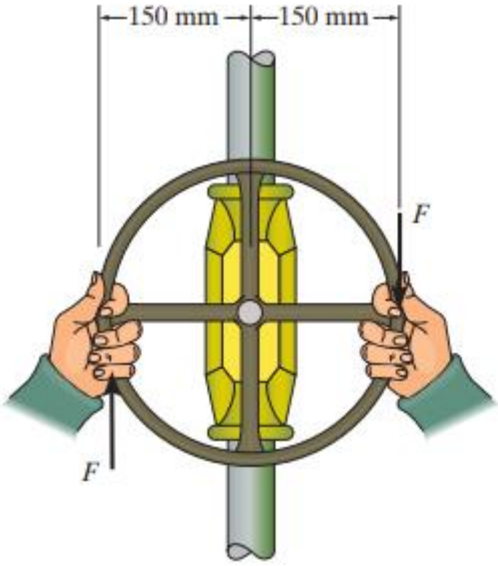
9. Determine the couple moment on the beam



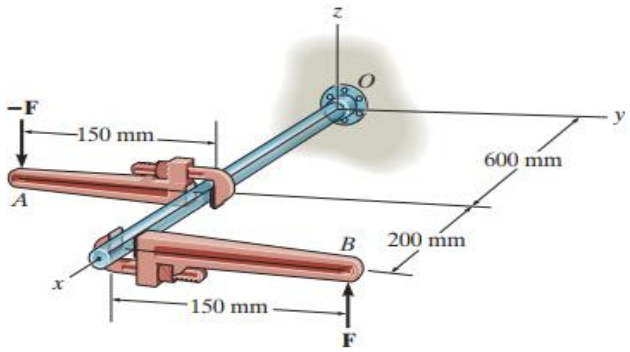
10. Determine the couple moment acting on the pipe assembly and express the result as a Cartesian vector



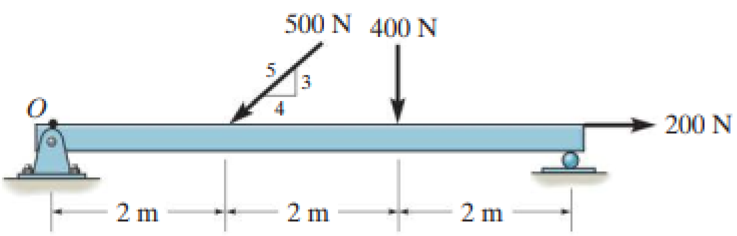
11. If the valve can be opened with a couple moment of 25 N.m, determine the required magnitude of each couple force which must be applied to the wheel



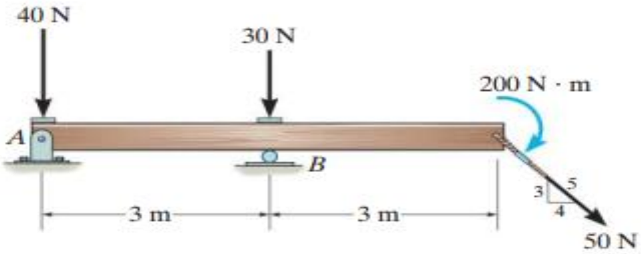
12. If the couple moment acting on the pipe has a magnitude of 300 N.m, determine the magnitude *F* of the forces applied to the wrenches.



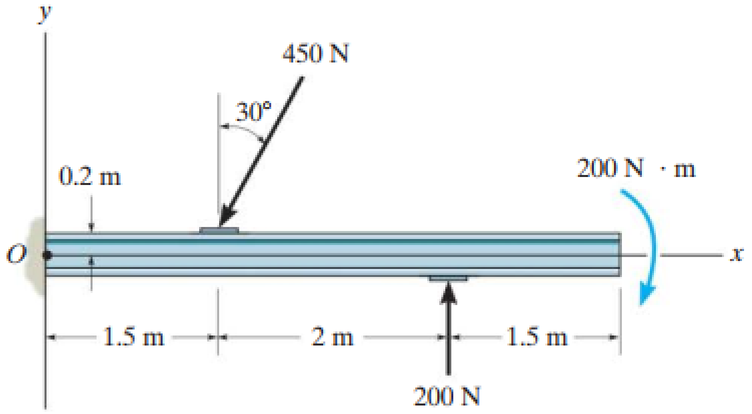
13. Determine the x and y component of the resultant force and the resultant couple moent t point O in the figure below



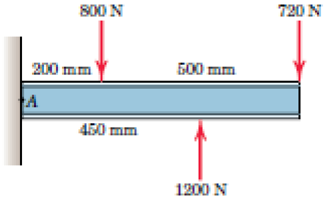
14. Replace the loading system by an equivlent resultant force and couple moment at point A



15. Replace the loading system acting on the bem by an equivalent force and couple moment at point O



16. Reduce the given loading system to a force–couple system at point *A*. Then determine the distance *x* to the right of point *A* at which the resultant of the three forces acts.



17. The control lever is subjected to a clockwise couple of 80 N.m exerted by its shaft at *A* and is designed to operate with a 200-N pull as shown. If the resultant of the couple and the force passes through *A*, determine the proper dimension *x* of the lever.