ENGINEERING MECHANICS (STATICS

INTRODUCTION

Mechanics is a branch of the physical sciences deals with the state of rest or motion of bodies that are subjected to the action of forces.

Generally, this subject can be subdivided into three branches

i. Rigid-body mechanics
ii. Deformable-body mechanics
iii. Fluid mechanics.

This subject is focused on rigid-body mechanics. Rigid-body mechanics is essential for the design and analysis of many types of structural members, mechanical components, or electrical devices encountered in engineering. Rigid body mechanics is divided into two main parts, vis statics and dynamics. Statics deals with the equilibrium of bodies, that is, those that are either at rest or move with a constant velocity; whereas dynamics is concerned with the accelerated motion of bodies.

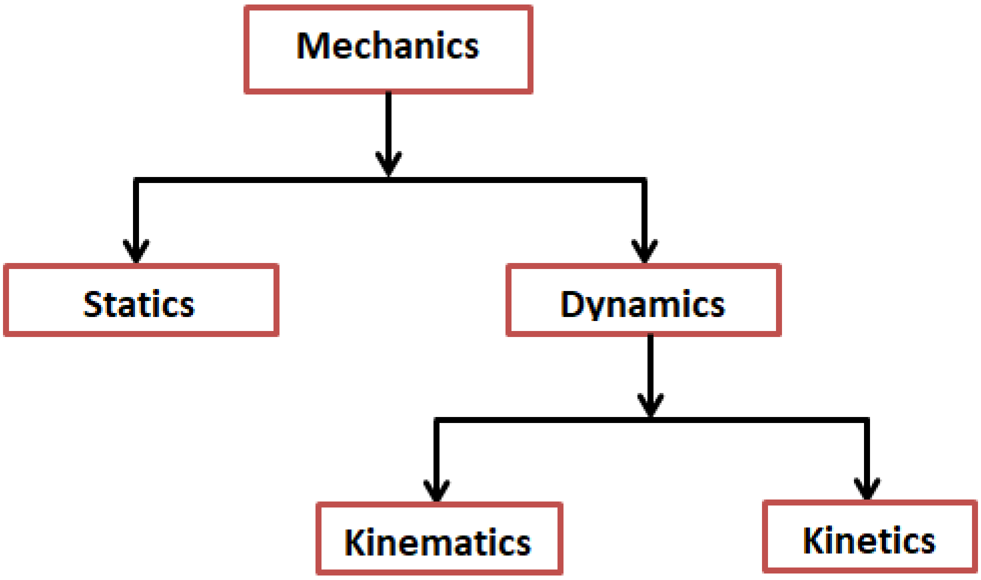


Figure: Branches of Mechanics

Fundamental Concept in Mechanics

Some fundamental concepts and principles relevant to the study of engineering mechanics are discussed below.

Basic Quantities: The four basic quantities used in mechanics are as follows:

- Length. Length is used to locate the position of a point in space and thereby describe the size of a physical system. Once a standard unit of length is defined, one can then use it to define distances and geometric properties of a body as multiples of this unit.
- Time. Time is conceived as a succession of events. Although the principles of statics are time independent, this quantity plays an important role in the study of dynamics.
- Mass. Mass is a measure of a quantity of matter that is used to compare the action of one body with that of another. This property manifests itself as a gravitational attraction

between two bodies and provides a measure of the resistance of matter to a change in velocity.

- Force. Generally, force is considered as a “push” or “pull” exerted by one body on another. This interaction can occur when there is direct contact between the bodies, such as a person pushing on a wall, or it can occur through a distance when the bodies are physically separated.

Idealizations: Models or idealizations are used in mechanics in order to simplify application of the theory. There are three important idealizations in mechanics.

- Particle. A particle has a mass, but a size that can be neglected. For example, the size of the earth is insignificant compared to the size of its orbit, and therefore the earth can be modeled as a particle when studying its orbital motion.
- Rigid Body. A rigid body can be considered as a combination of a large number of particles in which all the particles remain at a fixed distance from one another, both before and after applying a load.
- Concentrated Force. A concentrated force represents the effect of a loading which is assumed to act at a point on a body. We can represent a load by a concentrated force, provided the area over which the load is applied is very small compared to the overall size of the body.

Fundamental Principles in Mechanics

The study of elementary mechanics rests on six fundamental principles based on experimental evidence.

- **The Parallelogram Law for the Addition of Forces:** This states that two forces acting on a particle may be replaced by a single force, called their resultant, obtained by drawing the diagonal of the parallelogram which has sides equal to the given forces.
- **The Principle of Transmissibility:** This states that the conditions of equilibrium or of motion of a rigid body will remain unchanged if a force acting at a given point of the rigid body is replaced by a force of the same magnitude and same direction, but acting at a different point, provided that the two forces have the same line of action.
- **Newton’s Three Laws of Motion**: Engineering mechanics is formulated on the basis of Newton’s three laws of motion, the validity of which is based on experimental observation. These laws apply to the motion of a particle as measured from a non-accelerating reference frame. They may be briefly stated as follows.
    - First Law. A particle originally at rest, or moving in a straight line with constant velocity, tends to remain in this state provided the particle is not subjected to an unbalanced force.
    - Second Law. A particle acted upon by an unbalanced force (F) experiences an acceleration (a) that has the same direction as the force and a magnitude that is directly proportional to the force. If F is applied to a particle of mass (m), this law may be expressed mathematically as
      $F = ma$
    - Third Law. The mutual forces of action and reaction between two particles are equal, opposite, and collinear,

- **Newton’s Law of Gravitational Attraction:** the gravitational attraction between any two particles is given by

$$F = G\frac{m_1 m_2}{r^2}$$

F = force of attraction between the two particles

G = universal constant of gravitation, from experimental evidence G$= 66.73 \times 10^{-12}\ m^3/kg.s^2$

m_1, m_2 = mass of each particle

r = distance between the two particles

Weight. The weight of a particle is the force of attraction between the particle and the earth. For a particle having mass m_1 = m, assuming the earth to be a constant non-rotating sphere with uniform density having mass m_2 = M_e, then with the distance between the center of the earth and the particle being r. The expression for finding the weight (W) of a particle can be determined from Newton's law of gravitation as

$$F = G\frac{m_1 m_2}{r^2}$$

Therefore

$$W = G\frac{m.M_e}{r^2}$$

Let $g = G\frac{M_e}{r^2} =$ gravitational acceleration which is approximately equal to $9.81\ m/s^2$ at sea level and latitude of 45°

Then $W = mg$

Units of Measurement

A unit of measurement is a definite magnitude of a physical quantity. There are two main measurement systems

1. Metric System (International System, SI): this system is based on three main units meter-kilogram-second (mks system). SI is an abbreviation of French Expression (Syteme International) in English (International System). The common units in SI system are g, kg, ton, mm, m, N, kN, Kelvin, Celsius.
2. English System (British System or imperial System or US Customary System): this system is based on foot-pound-second. It is called the FPS system. The common units in British Systems are pound (lb) , slug, inch, feet, mile, Fahrenheit.

Prefixes: when a numerical quantity is either very large or very small, the units used to define its size may be modified by using a prefix. The common prefixes are listed below:

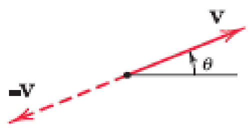
S/No	Multiplication Factor	Prefix	Symbol
1	10^{12}	terra	T
2	10^9	giga	G
3	10^6	mega	M
4	10^3	Kilo	k
5	10^2	hecto	h
6	10	deca	da
7	10^{-1}	deci	d
8	10^{-2}	centi	c
9	10^{-3}	milli	m
10	10^{-6}	micro	µ
11	10^{-9}	Nano	n
12	10^{-12}	pico`	P

Numerical Calculations
It is important that the answers to any problem be reported with both justifiable accuracy and appropriate significant figures
Dimensional Homogeneity
The terms of any equation used to describe a physical process must be dimensionally homogenous. For example,
$5\ m + 90\ cm$
$5\ m + 0.9\ m = 5.9\ m$
$or\ 500\ cm + 90\ cm = 590\ cm$

=

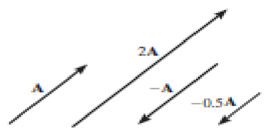
FORCE VECTORS

Two kinds of quantities are employed in engineering, they are: scalar quantities and vector quantities. Scalar quantities are the quantities with which only a magnitude is associated, for example, time, volume, density, speed, energy. Vector quantities are the quantities that possess both magnitude and. Examples of vector quantities include displacement, velocity, acceleration, force, moment, and momentum. Vectors representing physical quantities can be classified as free vector, sliding vector, or fixed vector. A vector can be represented graphically by an arrow. The length of the arrow represents the magnitude of the vector, and the angle θ between the vector and a fixed axis defines the direction of its line of action. The head or tip of the arrow indicates the sense of direction of the vector.

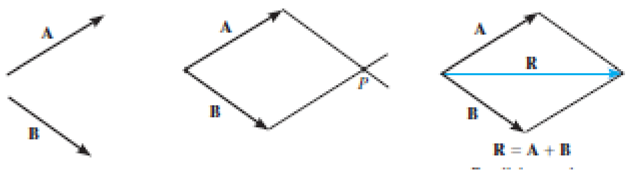


Vector Operations

i. Multiplication and Division of a Vector by a Scalar. If a vector is multiplied by a positive scalar, its magnitude is increased by that amount. Multiplying by a negative scalar will also change the directional sense of the vector.

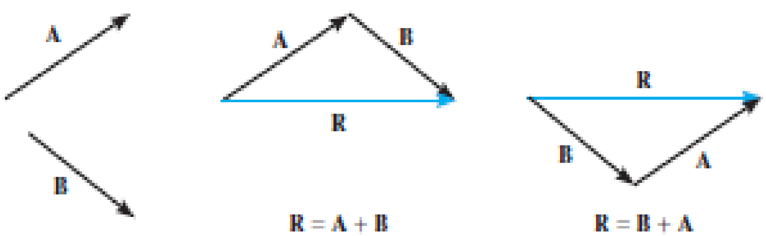


ii. Vector Addition. Two vectors A and B can be added together to form a resultant vector R = A + B by employing the parallelogram law. In doing this, vectors A and B are joined together at their tails and parallel lines are drawn from the head of each vector to intersect at a common point thereby forming a parallelogram which extends from the tail of vector A and B to the intersection point.

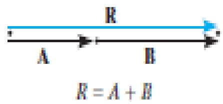


Also, vector B can be added to vector A using the triangle law which is a special case of parallelogram law. Here, the head of vector A is connected to the tail of vector B and the resultant extends from the tail of vector A to the head of vector B or head of vector B to the tail of vector A. The addition of vector is commutative, that is the vectors can be added in any order

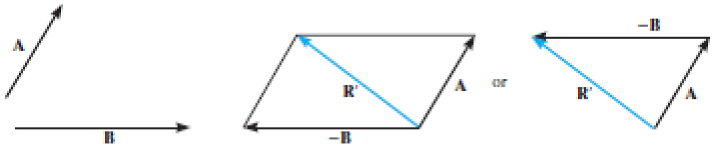
R = A + B = B +A.



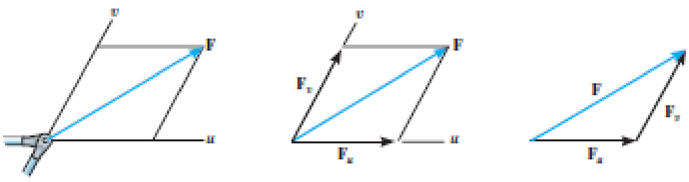
A special case arise when the two vectors are collinear (having the same line of action), the vector addition then turns to algebraic sum or scalar addition.



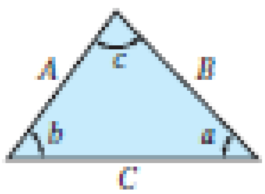
iii. Vector subtraction. The resultant difference of two vectors A and B is expressed as R' = A-B = A + (-B)



iv. Resolution of forces. A vector may be resolved into two components having known lines of action by using parallelogram law. If for example, a resultant force F is to be resolved into two components along u and v direction. A parallelogram is constructed by drawing lines to directions u and v from the head of R. These lines will intersect the v and u axes to form a parallelogram. The force component F_u and F_v are then established by joining the tail of F to the intersection points on the u and v axes. This parallelogram can be reduced to a triangle to represent the triangle rule and the laws of sine and cosine applied to determine the unknown magnitudes of the components.



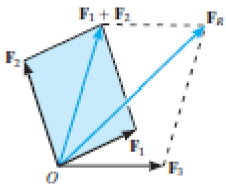
Sine and Cosine Law



Cosine Law: $C = \sqrt{A^2 + B^2 - 2AB\cos c}$

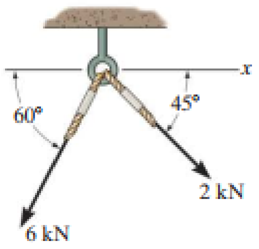
Sine law: $\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$

v. Addition of several concurrent forces. In cases where more than two forces needs to be added successive application of the parallelogram law is used.

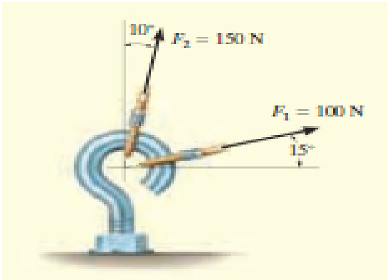


Example

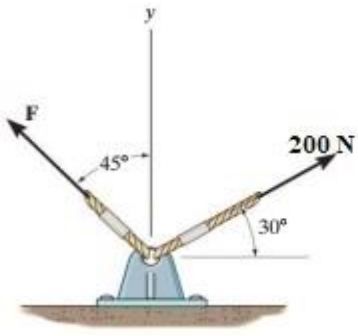
1. Determine the magnitude of the resultant force acting on the screw eye and its direction measured clockwise from the x axis



2. The screw eye thread shown below is subjected to two forces F_1 and F_2, determine the magnitude and direction of the resultant force.

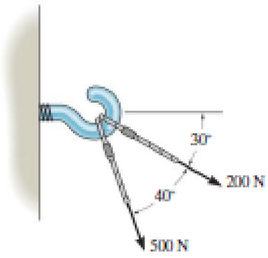


3. Determine the magnitude of the component force F and the magnitude of the resultant F_R. If F_R is directed along the positive y -axis

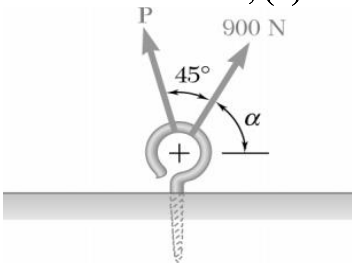


Practice Questions

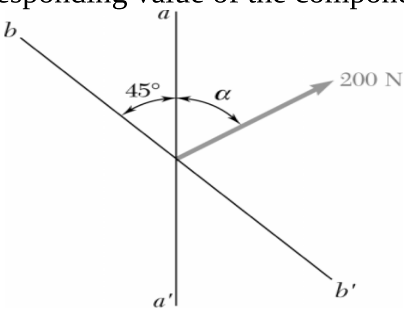
1. Two forces act on the hook. Determine the magnitude of the resultant force.



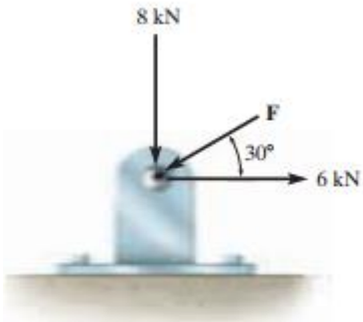
2. Two forces are applied as shown to a hook support. Using trigonometry and knowing that the magnitude of P is 600 N, determine (*a*) the required angle α if the resultant R of the two forces applied to the support is to be vertical, (*b*) the corresponding magnitude of R.



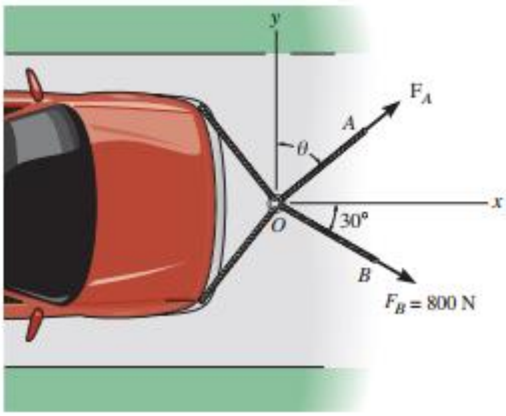
3. The 200-N force is to be resolved into components along lines *a-a′* and *b-b′*. (*a*) Determine the angle α using trigonometry knowing that the component along *a-a′* is to be 150 N. (*b*) What is the corresponding value of the component along *b-b′*?



4. Determine the magnitude of force F so that the resultant F_R of the three forces is as small as possible. What is the minimum magnitude of F_R?



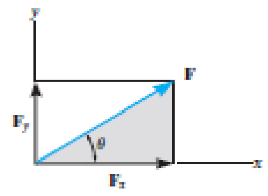
5. Determine the magnitude and direction, measured counterclockwise from the positive *x* axis, of the resultant force acting on the ring at *O*, if F_A = 750 N and θ = 45°.



Resolution of Forces into Rectangular Component

When a force is resolved into two components along the x and y axes, the components are then called rectangular components. For analytical work we can represent these components by, using either scalar or Cartesian vector notation.

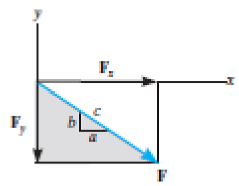
a. **Scalar notation.** The rectangular components of force **F** shown in the figure below are determined from trigonometry as



$$\cos\theta = \frac{F_x}{F} \xrightarrow{gives} F_x = F\cos\theta$$

$$\sin\theta = \frac{F_y}{F} \xrightarrow{gives} F_y = F\sin\theta$$

If the direction of F is defined a slope instead of the angle θ as shown, then by similar triangles

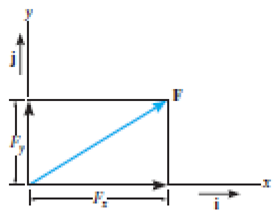


$$\frac{F_x}{F} = \frac{a}{c} \xrightarrow{gives} F_x = F\left(\frac{a}{c}\right)$$

Also,

$$\frac{F_y}{F} = \frac{b}{c} \xrightarrow{gives} F_y = -F\left(\frac{b}{c}\right)$$

b. **Cartesian vector notation.** It is also possible to represent the x and y components of a force in terms of the Cartesian unit vectors i and j. these unit vectors i and j are dimensionless and are used to designate the directions of the components. The force F can be expressed in Cartesian vectors as



$$\boldsymbol{F} = F_x i + F_y j$$

Where F_x and F_y are magnitudes which are always positive and i and j are unit vectors which may be positive or negative.

Resultant of Coplanar Forces

Any of the scalar notation or Cartesian vector notation methods can be employed to determine the resultant of several coplanar forces. Consider for example a system of coplanar forces in the figure below

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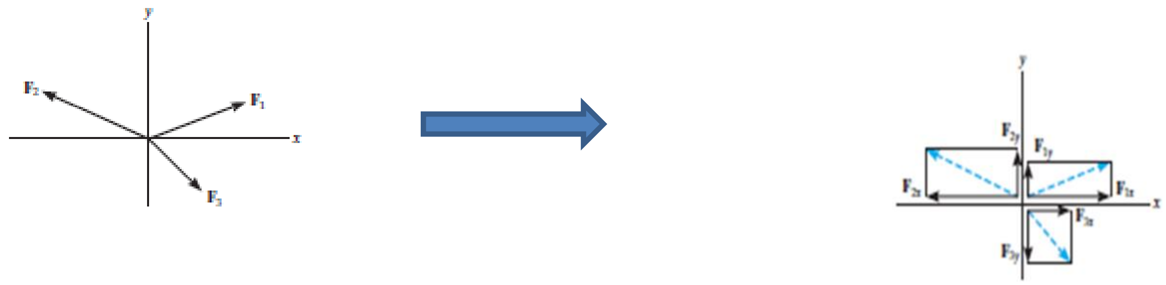


Scalar Notation Method:

$\overset{+}{\rightarrow} F_{Rx} = F_{1x} + F_{2x} + F_{3x} = \sum F_x$

$+\uparrow\ F_{Ry} = F_{1y} + F_{2y} + F_{3y} = \sum F_y$

$F_R = \sqrt{(F_{Rx})^2 + \left(F_{Ry}\right)^2}$

$\theta = \tan^{-1}\frac{F_{Ry}}{F_{Rx}}$

Where F_{Rx} *and* F_{Ry} are the resultants in the x and y directions respectively and θ is the direction of F_R

Cartesian Vector Notation Method:

$\boldsymbol{F_1} = F_1xi + F_1yj$

$\boldsymbol{F_2} = F_2xi + F_2yj$

$\boldsymbol{F_3} = F_3xi + F_3yj$

$\boldsymbol{F_R} = F_1 + F_2 + F_3$

$= F_1xi + F_1yj + F_2xi + F_2yj + F_3xi + F_3yj$

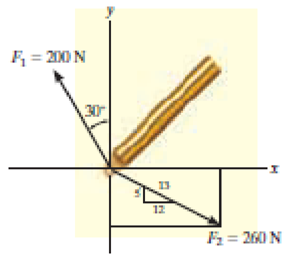
$= (F_1x + F_2x + F_3x)i + (F_1y + F_2y + F_3y)j$

$= (F_{Rx})i + \left(F_{Ry}\right)j$

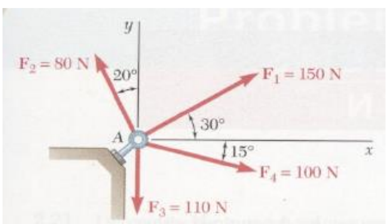
The magnitude and the direction of the resultant force F_R can be determined as previously explained in scalar notation method

Examples

1. Determine the *x* and *y* components of F_1 and F_2 acting on the boom shown in the figure below. Express each force as a Cartesian vector. Also find the resultant of the forces acting on the boom

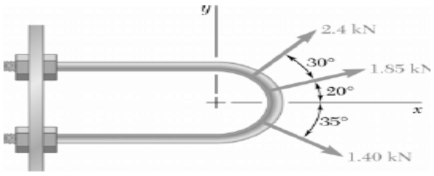


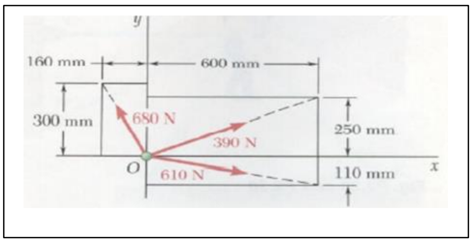
2. Four forces act on bolt A as shown in the figure below. Determine the resultant of the forces on the bolt.

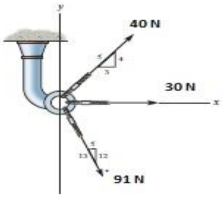


Practice Problems

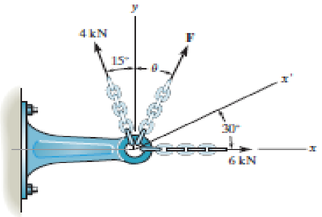
1. Determine the resultant of the forces shown in the figures below



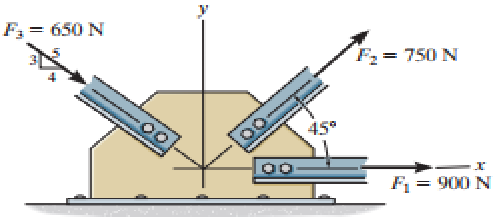




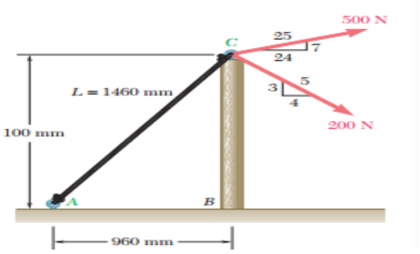
2. If F = 5 kN and θ = 30°, determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive *x* axis.



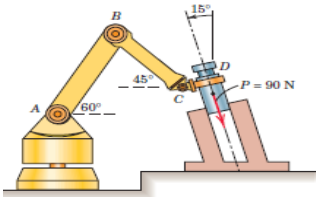
3. Determine the magnitude of the resultant force acting on the plate and its direction, measured counterclockwise from the positive *x* axis.



4. Knowing that the tension in rope *AC* is 365 N, determine the resultant of the three forces exerted at point *C* of post *BC*



5. In the design of the robot to insert the small cylindrical part into a close-fitting circular hole, the robot arm must exert a 90-N force P on the part parallel to the axis of the hole as shown. Determine the components of the force which the part exerts on the robot along axes (a) parallel and perpendicular to the arm AB, and (b) parallel and perpendicular to the arm BC.

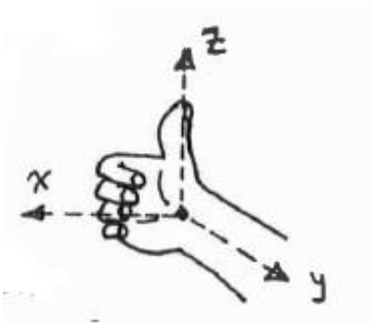


FORCES IN THREE DIMENSIONS

The problems of forces in three dimensions are greatly simplified when the forces are represented in Cartesian vector form

Right-handed Coordinate System

The right hand rule is used to develop the theory of vector algebra. In this rule, the thumb points in the positive z axis when the fingers are curled about this axis and directed from the positive x-axis to the positive y-axis.



Rectangular Components of a Vector

A vector **A** may have one, two or three rectangular components along the x, y, z axes depending on how the vector is oriented relative to the axes. We can resolve the vector shown in the figure by employing parallelogram law as

$\mathbf{A} = \mathbf{A'} + \mathbf{A_z}$

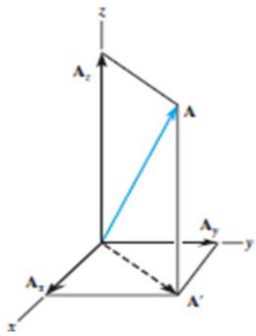
But $\mathbf{A'} = \mathbf{A_x} + \mathbf{A_y}$

Therefore combining the two equations above we have

$\mathbf{A} = \mathbf{Ax} + \mathbf{Ay} + \mathbf{A_z}$

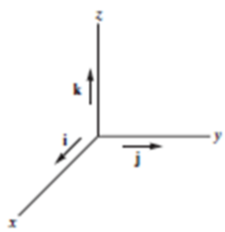
The magnitude of vector **A** can be determined using Pythagorean theorem as

$/A/= \sqrt{A_x^2 + A_y^2 + A_z^2}$



Cartesian Unit Vectors

In three dimensions, the set of Cartesian unit vectors, i, j, k, is used to designate the directions of the x, y, z axes, respectively. The sense (or arrowhead) of these vectors is represented analytically by a positive or negative sign, depending on whether they are directed along the positive or negative x, y, or z axes.

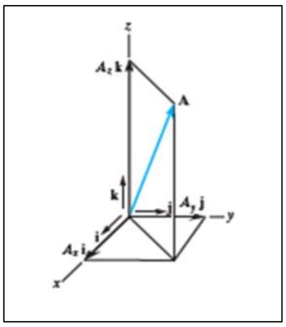


Cartesian Vector Representation

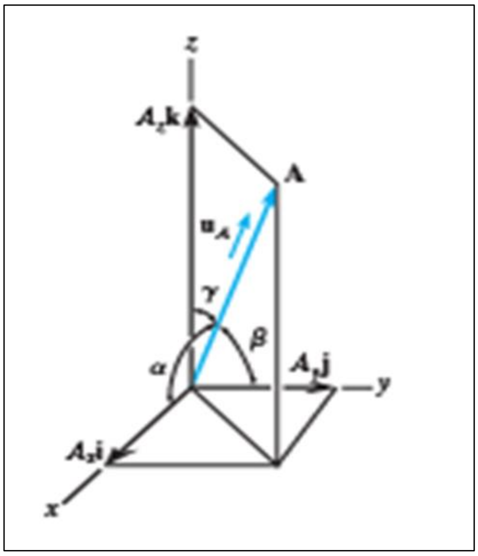
The representation of a vector **A** in Cartesian vector form is

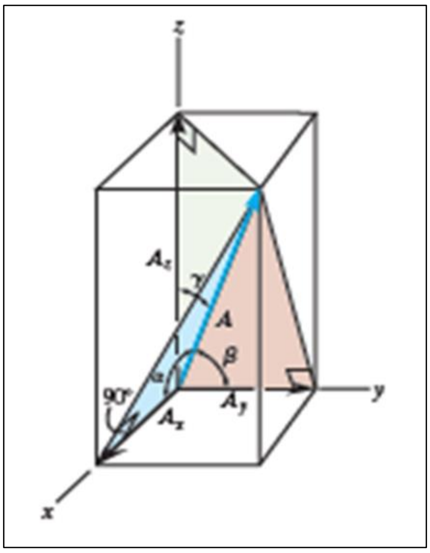
$\mathbf{A} = \mathrm{A_x i} + \mathrm{A_y j} + \mathrm{A_z k}$

The magnitude and direction of each component vector can be separated to simplify the operations of the vector algebra, particularly in three dimensions



Direction Cosines of Cartesian Vectors
The direction of vector **A** can be defined by the coordinate direction angles α (alpha), β (beta), and γ (gamma), measured between the tail of vector **A** and the positive x, y, z axes respectively provided they are located at the tail of vector **A** as shown in figure (a) below**.** To determineα, β, γ consider the projection of **A** onto the *x, y, z* axes as shown in the figure (b).





From shaded right triangles shown we have
$\cos\alpha = \frac{A_x}{A}$; $\cos\beta = \frac{A_y}{A}$; $\cos\gamma = \frac{A_z}{A}$ *

These are known as the direction cosines of A. The coordinate direction angle α, β, and γ, can then be determined from the inverse cosines. These direction cosines can be easily obtained by forming a unit vector u_A in the direction of vector **A.** If vector **A** is expressed in Cartesian vector form, we have
$\mathbf{A} = A_x i + A_y j + A_z k$
The unit vector u_A will have a magnitude of one and be dimensionless provided vector **A** is divided by its magnitude, that is,
$\boldsymbol{u_A} = \frac{\mathbf{A}}{A} = \frac{A_x}{A}\mathbf{i} + \frac{A_y}{A}\mathbf{j} + \frac{A_z}{A}\mathbf{k}$ **

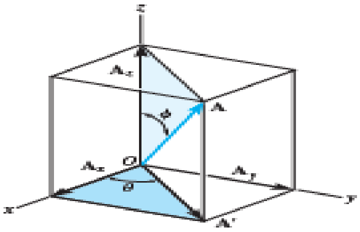
From above equation, it can be seen that the i, j, k components of u_A represents the direction cosines, therefore comparing * and **
$u_A = \cos\alpha\, i + \cos\beta\, j + \cos\gamma\, k$

Since the magnitude of a vector is equal to the positive square root of the sum of the squares of the magnitudes of its components, and u_A has a magnitude of one, then from the above equation an important relation among the direction cosines can be formulated as

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

Therefore the vector expression in terms of angles is
$\mathbf{A} = \mathbf{A}\cos\boldsymbol{\alpha}\,\boldsymbol{i} + \boldsymbol{A}\cos\boldsymbol{\beta}\,\boldsymbol{j} + \boldsymbol{A}\cos\boldsymbol{\gamma}\,\boldsymbol{k}$

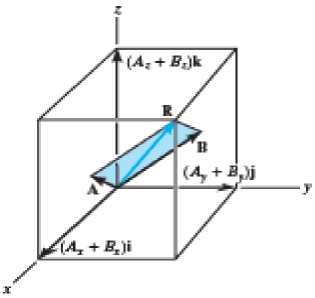
Sometimes, the direction of A can be specified using two angles, namely, a transverse angle (θ) and an azimuth angle ϕ (phi), such as shown in figure below. The components of vector **A** can then be determined by applying the principle of trigonometry to the right hand triangle at the top as



$A_z = A \cos \phi$ and $A' = A \sin \phi$

In the right angle triangle at the base of the figure
$A_x = A' \cos \theta = A \sin \phi \cos \theta$ and $A_y = A' \sin \theta = A \sin \phi \sin \theta$
Therefore vector **A** can be written in Cartesian form as
$\mathbf{A} = \mathbf{A \sin \phi \cos \theta}\, \boldsymbol{i} + \boldsymbol{A} \mathbf{\sin \phi \sin \theta} + \boldsymbol{A} \mathbf{\cos \phi}\, \boldsymbol{k}$

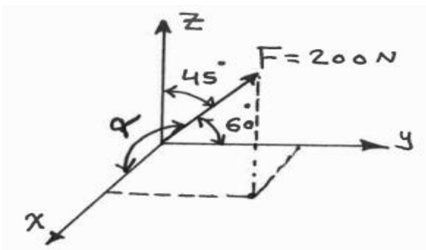
Addition of Cartesian Vectors in 3D
The addition (or subtraction) of two or more vectors is greatly simplified if the vectors are expressed in terms of their Cartesian components. For example, if A = Ax i + Ay j + Az k and
B = B_xi + B_y j + B_zk, as shown in the figure below



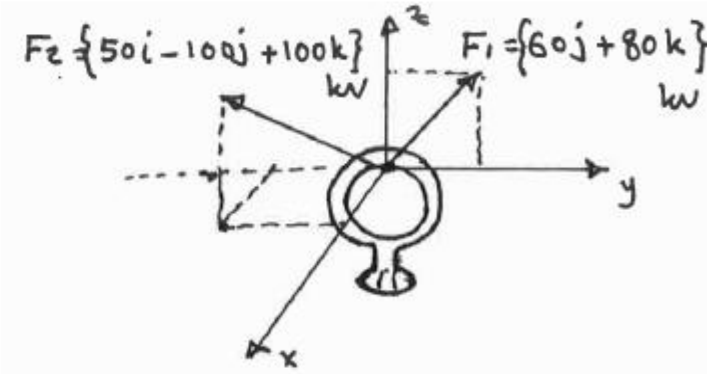
Then the resultant vector, R, has components which are the scalar sums of the i, j, k components of A and B, i.e.,
$R = A + B = (A_x + B_x)i + (A_y + B_y)j + (A_z + B_z)\,k$
The equation above can be generalized and applied to a system of several concurrent forces, then the force resultant is the vector sum of all the forces in the system and can be written as
$F_R = \sum F = \sum F_x\, i + \sum F_y\, j + \sum F_z\, k$
$\sum F_x, \sum F_y, \sum F_z$ represents the algebraic sums of the respective x, y, z or i, j, k components of each force in the system.

Example

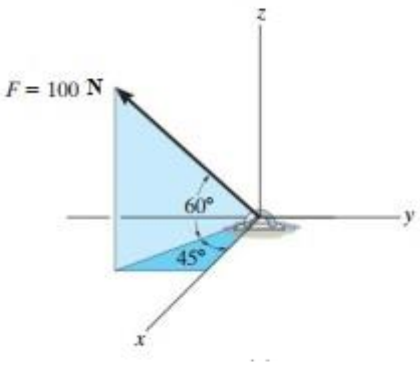
i. Express the fore F as a Cartesian unit vector



ii. Determine the magnitude and direction angles of the resultant force acting on the ring below

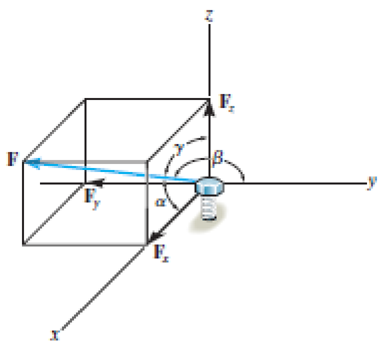


iii. Express the force shown in the figure below as a Cartesian vector and find the direction angles

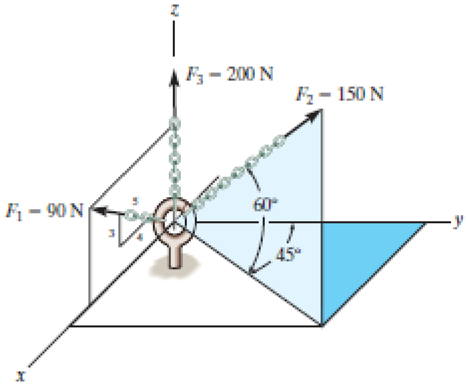


Practice Questions

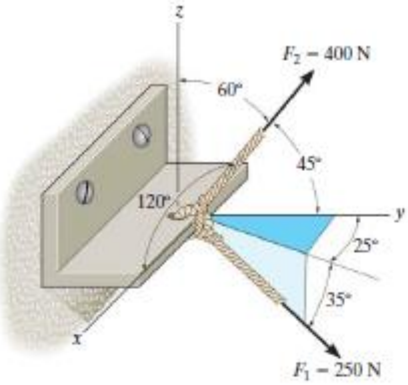
1. The bolt is subjected to the force **F**, which has components acting along the *x, y, z* axes as shown. If the magnitude of F is 80 N, and α = 60° and Υ= 45°, determine the magnitudes of its components.



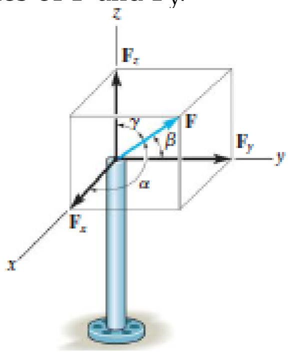
2. Determine the magnitude and coordinate direction angles of the resultant force, and sketch this vector on the coordinate system.



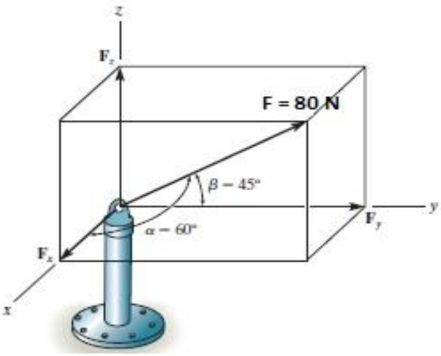
3. The bracket is subjected to the two forces shown. Express each force in Cartesian vector form and then determine the resultant force F_R. Find the magnitude and coordinate direction angles of the resultant force.



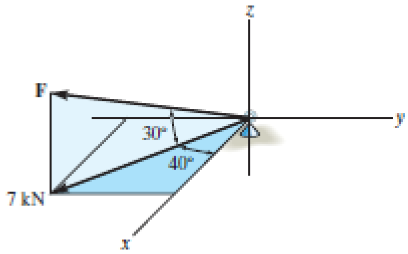
4. The pole is subjected to the force F which has components $F_x = 1.5$ kN and $F_z = 1.25$ kN. If $\beta = 75°$, determine the magnitudes of F and F_y.



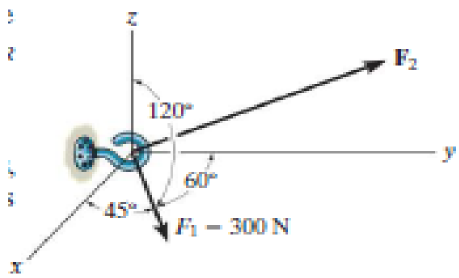
5. The force F has a magnitude of 80 N and acts within the octant shown. Determine the magnitudes of the *x*, *y*, *z* components of F.



6. Determine the magnitude and coordinate direction angles of the force F acting on the support. The component of F in the *x*–*y* plane is 7 kN.

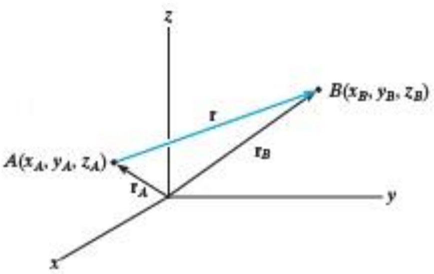


7. Two forces act on the hook shown below. Specify the magnitude of F_2 and its coordinate direction angles so that the resultant force F_R acts along the positive *y* axis and has a magnitude of 800 N.



Position Vector

A position vector r is defined as a fixed vector which locates a point in space relative to another point.



From the figure above r can be determined from vector addition as

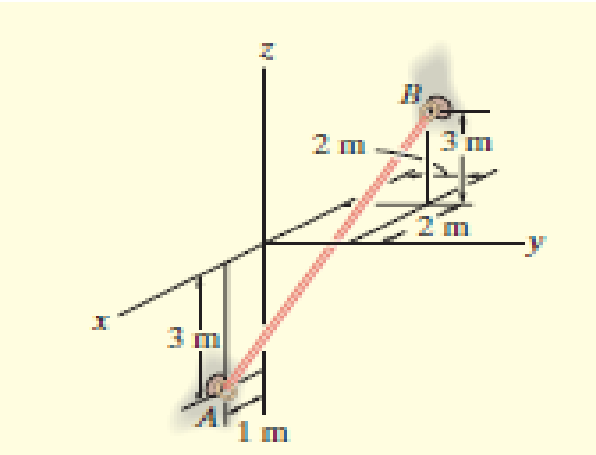
$$\boldsymbol{r_B} = \boldsymbol{r_A} + \boldsymbol{r}$$

$$\boldsymbol{r} = \boldsymbol{r_B} - \boldsymbol{r_A}$$

$$\boldsymbol{r} = (x_B i + y_B j + z_B k) - (x_A i + y_A j + z_A k)$$
$$\boldsymbol{r} = (x_B - x_A)i + (y_B - y_A)j + (z_B - z_A)k$$

Note that the position vector is obtained by subtracting the coordinates of the tail of vector r from the coordinates of the head

Example
An elastic rubber band is attached to points A and B as shown below. Determine its length and its direction measured from A toward B.



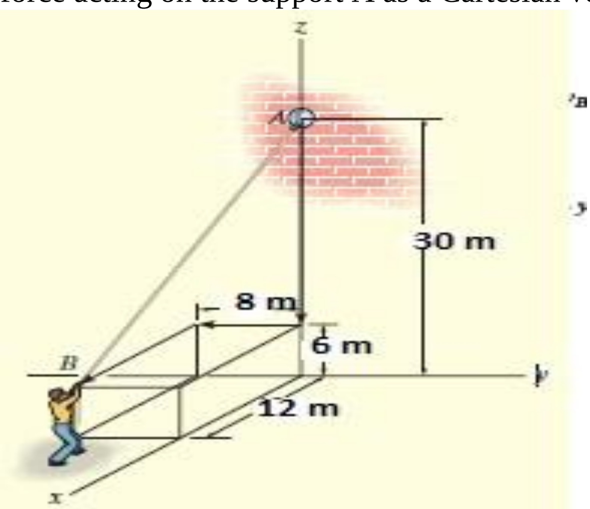
Force Vector Directed Along a Line

To represent the force directed along an element AB, the unit vector u is multiplied by the magnitude of the force.

The unit vector $u_F = \frac{\boldsymbol{r}}{/r/}$

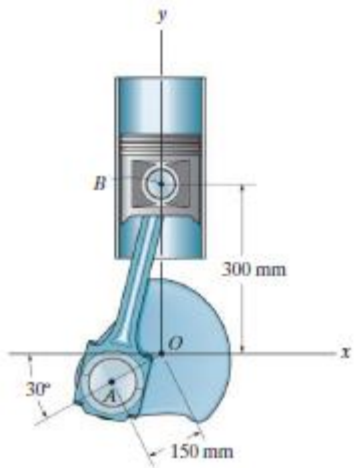
$$\mathbf{F} = \mathrm{F}\mathrm{u}_\mathrm{F} = \mathrm{F}\frac{\mathbf{r}}{/\mathrm{r}/}$$

Example
The man shown in the figure below pulls on the cord with a force of 70 N. Represent this force acting on the support A as a Cartesian vector and determine its direction.

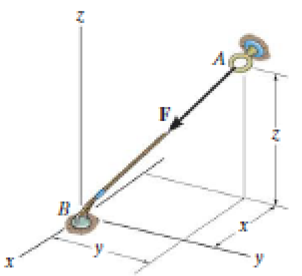


Practice Problems

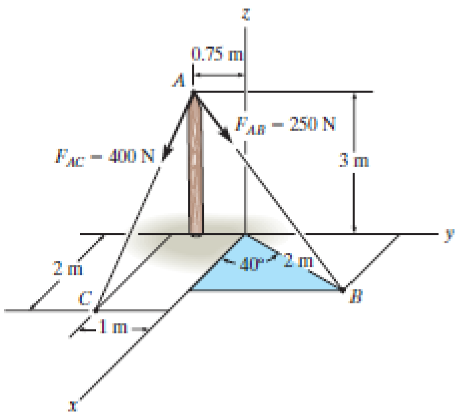
1. Determine the length of the connecting rod *AB* by first formulating a Cartesian position vector from *A* to *B* and then determining its magnitude



2. If F = (350i - 250j - 450k) N and cable AB is 9 m long, determine the x, y, z coordinates of point A.



3. Express each of the forces in Cartesian vector form and determine the magnitude and coordinate direction angles of the resultant force



Dot Product

The dot product is a multiplication of two vectors. It is defined as

$\mathbf{A}.\mathbf{B} = \mathrm{AB}\cos\theta$

Where **A** and **B** are vectors and θ is the angle between their tail in which$0 \leq \theta \leq 180$

The dot product is often referred to as the scalar product of vectors since the result is a scalar and not a vector.

Law of Operation for Dot Product

1. Commutative law A.B = B.A
2. Multiplication by a scalar; $\alpha(A.B) = (\alpha A).B = A.(\alpha B)$
3. Distributive law: A. (B+D) = (A.B) + (A.D)

Dot Product in Cartesian Vector Form

$A = A_x i + A_y j + A_z k$

$B = B_x i + B_y j + B_z k$

$A.B = \left(A_x i + A_y j + A_z k\right).\left(B_x i + B_y j + B_z k\right)$

$= A_x B_x(i.i) + A_x B_y(i.j) + A_x B_z(i.k)$

$+A_y B_x(j.i) + A_y B_y(j.j) + A_y B_z(j.k)$

$+A_z B_x(k.i) + A_z B_y(k.j) + A_z B_z(k.k)$

Note that

$i.i = j.j = k.k = (1)(1)\cos 0 = 1$

$i.k = i.j = j.k = (1)(1)\cos 90 = 0$

Therefore,

$A.B = A_x B_x + A_y B_y + A_z B_z$

Application of Dot Product

Dot product has two main applications in mechanics especially in three dimensional problems

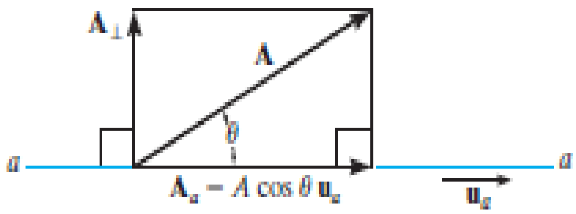
1. It is used in finding the angle forms between two intersecting lines or vectors

$\mathbf{A}.\mathbf{B} = \mathrm{AB}\cos\theta$

$\cos\theta = \frac{\mathbf{A.B}}{AB}$, $\quad 0 \le \theta \le 180$

2. Finding components that are perpendicular or parallel to a line or vector

   If u_a is a unit vector in the direction of a



$A_a = A\cos\theta$

But $\mathbf{A}.\mathbf{u_a} = A u_a \cos\theta = A\cos\theta$ $\quad$ [since $u_a = 1$]

$A_a = \mathbf{A}.\mathbf{u_a}$

The perpendicular component A_p is determined as follows

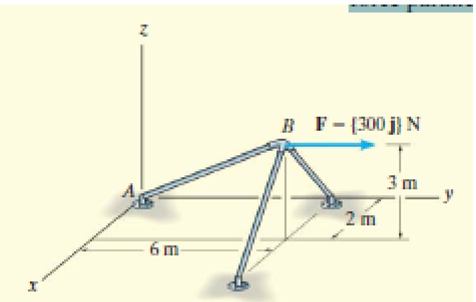
$\mathbf{A} = \mathbf{A_a} + \mathbf{A_p}$

$\mathbf{A_p} = \mathbf{A} - \mathbf{A_a}$

Note that A_a is the projection of A along 'a' axis and A_p is the projection of A along p axis

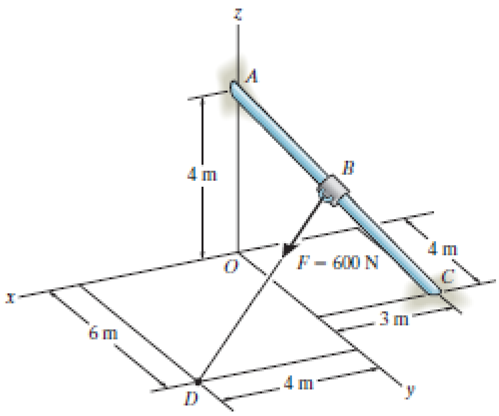
Example

1. The frame shown below is subjected to a horizontal force F = {300j} N. Determine the magnitudes of the components of this force parallel and perpendicular to member AB

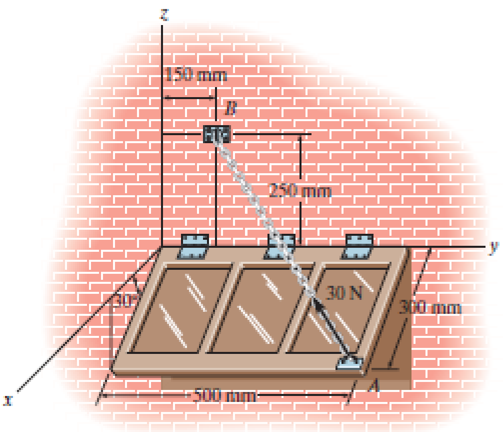


Practice Problems

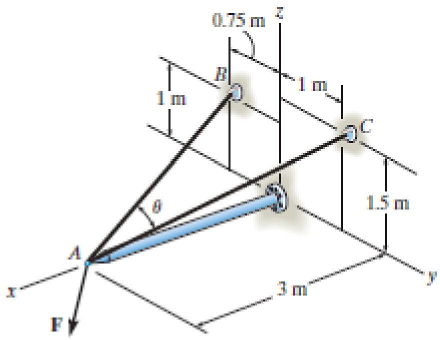
1. Express the force F in Cartesian vector form if it acts at the midpoint *B* of the rod.



2. The window is held open by cable AB. Determine the length of the cable and express the 30-N force acting at A along the cable as a Cartesian vector.



3. Determine the angle θ between cables AB and AC


0.75 m
z
B
1 m
1 m
C
1.5 m
θ
A
x
y
3 m
F