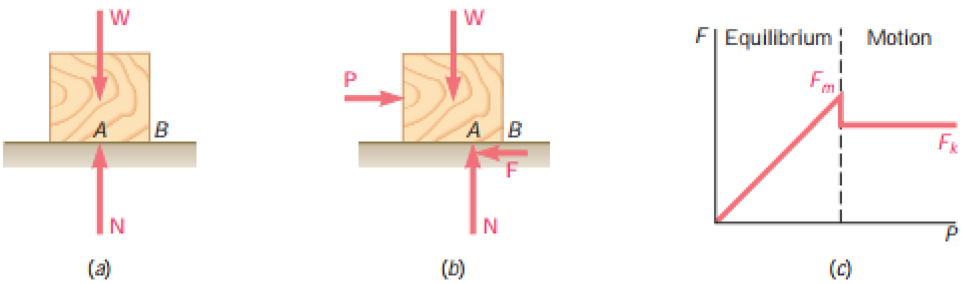
FRICTION

There are two types of friction: dry friction (Coulomb friction) and fluid friction. Fluid friction develops between layers of fluid moving at different velocities. Fluid friction is of great importance in problems involving the flow of fluids through pipes and orifices or dealing with bodies immersed in moving fluids. It is also basic in the analysis of the motion of lubricated mechanisms. Such problems are considered in texts on fluid mechanics. The present study is limited to dry friction, i.e., to problems involving rigid bodies which are in contact along non-lubricated surfaces.
When a body slides or tends to slide on another body, the force that is tangent to the contact surface which resists the motion or the tendency towards motion of one body relative to the other is friction. When two bodies are in contact and assumed to be smooth, the reaction of one body on the other is a force normal to the contact surface. In actual practice, the contact surface is not smooth and the reaction is resolved in two components, one perpendicular and the other tangent to the contact surface. The component tangent to the surface is called the frictional force or the friction.

Laws of Dry Friction

Consider a block having weight W placed on a horizontal plane surface as shown in figure (a) below. The forces acting on the block are its weight W and the reaction of the surface. Since the weight has no horizontal component, the reaction of the surface has no horizontal component; the reaction is therefore normal to the surface and represented by N. Let a horizontal force P now be applied to the block as shown in figure (b). If the force P is small, the block will not move; some other horizontal force therefore must exist which balances P. the other force is the static friction force F. If the force P is further increased, the frictional force F also increases, continuing to oppose p, until it reaches a certain maximum limit F_m shown in figure (c). With further increase in P, the friction force cannot balance it again, therefore, the block starts sliding. Immediately the block is set in motion, the magnitude of F drops from F_m to F_k. Thereafter, the block continues to slide with increasing velocity while the friction force denoted by F_k remains approximately constant



Coefficient of Friction

Experimental evidence shows that the maximum value F_m of static friction force is proportional to the normal component N of the reaction of the surface

$F_m = \mu_s N$
Where μ_s is the coefficient of static friction.
Also, the magnitude F_k of the kinetic fiction may be expressed as

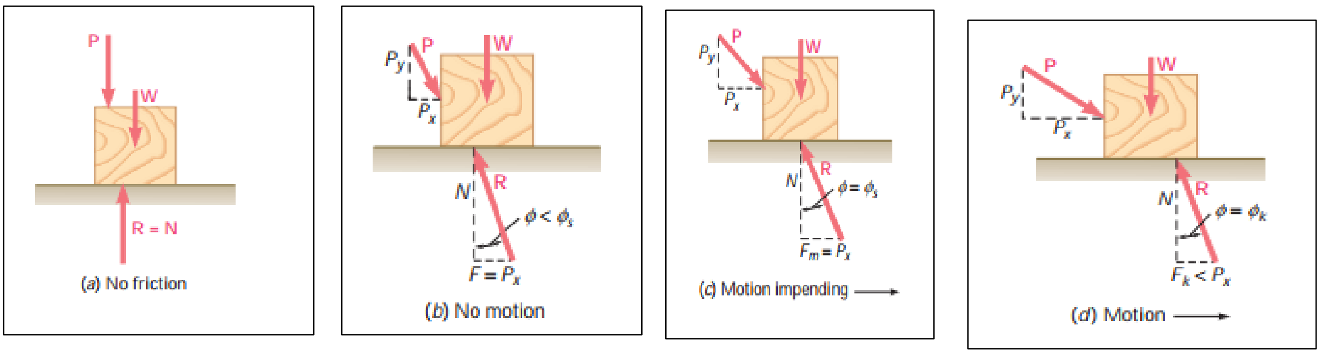
$F_k = \mu_k N$

Where μ_k is the coefficient of kinetic friction. The coefficients of friction μ_s and μ_k do not depend upon the area of the surfaces in contact. Both coefficients however depend strongly on the nature of the surfaces in contact. The table below shows the values of coefficient of static friction obtained by experiments on dry surfaces.

Contact Materials	Coefficient of Static Friction
Steel on steel	0.4 – 0.8
Wood on wood	0.3 – 0.7
Wood on metal	0.2 – 0.6
Rubber on concrete	0.6 – 0.8
Rubber on ice	0.05 – 0.2
Metal on ice	0.03 – 0.05

Angle of Friction

Sometimes, it is convenient to replace the normal force N and the friction force F by their resultant R. Consider a block of weight W resting on a horizontal plane surface. If no horizontal force is applied to the block, the resultant R reduces to the normal force N (figure a). However, if the applied force P has a horizontal component P_x which tends to move the block, the force R will have a horizontal component F which will form an angle ϕ with the normal to the surface as shown in Figure (b). If P_x is increased until motion becomes impending, the angle between R and the vertical increases and reaches a maximum as shown in figure (c). This value is called the angle of static friction and is denoted by ϕ_s. From the geometry of figure (c)



$$\tan \varnothing_s = \frac{F_m}{N} = \frac{\mu_s N}{N}$$

$$\tan \varnothing_s = \mu_s$$

If the motion actually takes place, the magnitude of the friction force drops to F_k; similarly, the angle ϕ between R and N deceases to a lower value ϕ_k, called the angle of kinetic friction as shown in figure (d). From the geometry of figure (d) we have

$$\tan \varnothing_k = \frac{F_k}{N} = \frac{\mu_k N}{N}$$

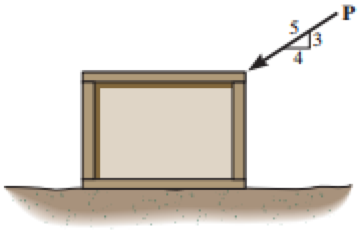
$$\tan \varnothing_k = \mu_k$$

The angle of friction can be used in the analysis of certain types of problems. Consider a block resting on a board which is subjected to no other force apart from its weight W and the reaction R of the board. The board can be given any desired inclination. If the board is horizontal, the force R exerted by the board on the block is perpendicular to the board and balances the weight W as shown in figure (a) below. If the board is given a small inclination θ, the force R will deviate from the perpendicular to the board by the angle θ and will keep balancing (figure b); it will have a normal component N having a magnitude $N = W\cos\theta$ and a tangential component $F = W\sin\theta$.

It the angle of inclination is gradually increased, it will get to a point when the motion becomes impending. At that time, the angle between R and the normal will have reached its maximum value $\varnothing_s$ (figure c). The value of the angle of inclination corresponding to impending motion is called the angle of repose. The angle of repose is equal to the angle of static friction $\varnothing_s$. If the angle of inclination θ is further increased, motion starts and the angle between R and the normal drops to the lower value $\varnothing_k$ (figure d). The reaction R is not vertical any more, and the forces acting on the block are unbalanced.

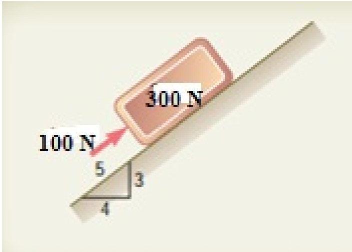
Examples

1. Determine the friction developed between the 50 kg crate and the ground if (a) P = 200 N and (b) P = 400. The coefficient of static and kinetic friction between the crate and the ground are $\mu_s = 0.3$ and $\mu_k = 0.2$



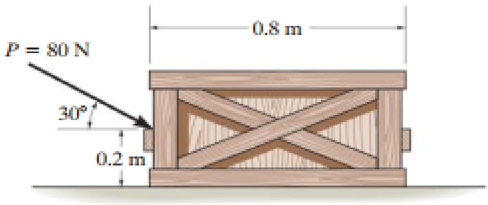
2. The 100 N force act as shown in the figure below on the 300 N block placed on an inclined plane. The coefficients of friction between the block and the plane

are μ_s = 0.25 and μ_k = 0.20. Determine whether the block is in equilibrium, and find the value of the friction force

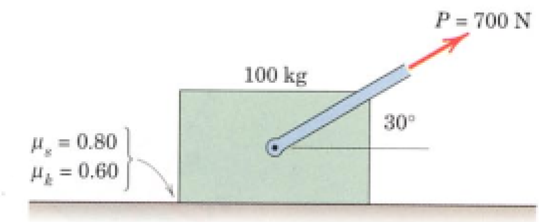


Practice Problem

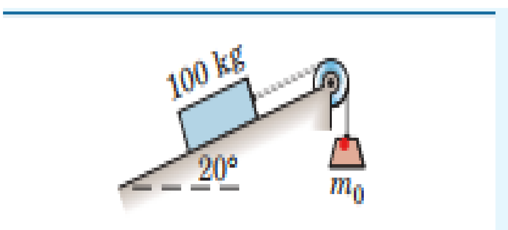
1. The crate shown in the figure has a mass of 20 kg, determine if it remains in equilibrium. The coefficient of static friction μ_s = 0.3



2. The 700 N force is applied to the 100 kg block, which is stationary before the force is applied. Determine the magnitude and direction of the friction force F exerted by the horizontal surface on the block

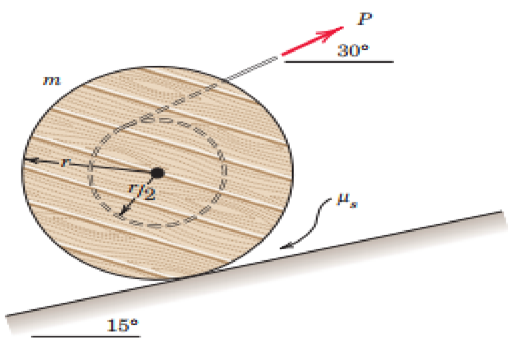


3. Determine the range of values which the mass m_o may have so that the 100 kg block shown in the figure will neither start moving up the plane nor slip down the plane. The coefficient of static friction for the contact surface is 0.30

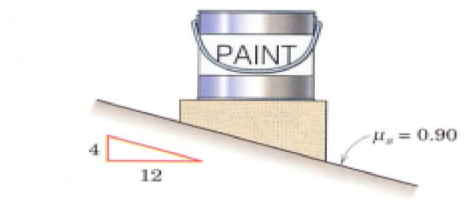


4. Determine the minimum coefficient of static friction μ_s = 0.3 which will allow the drum with fixed inner hub to be rolled up the incline at a steady speed without slipping. What are the corresponding values of the force *P* and the friction force

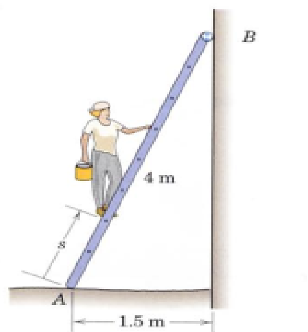
F?



5. The 1.2 kg wooden block is used for level support of the 9 kg can of paint. Determine the magnitude and direction of (a) the friction force exerted by the roof surface on the wooden block (b) the total force exerted by the roof surface on the wooden block



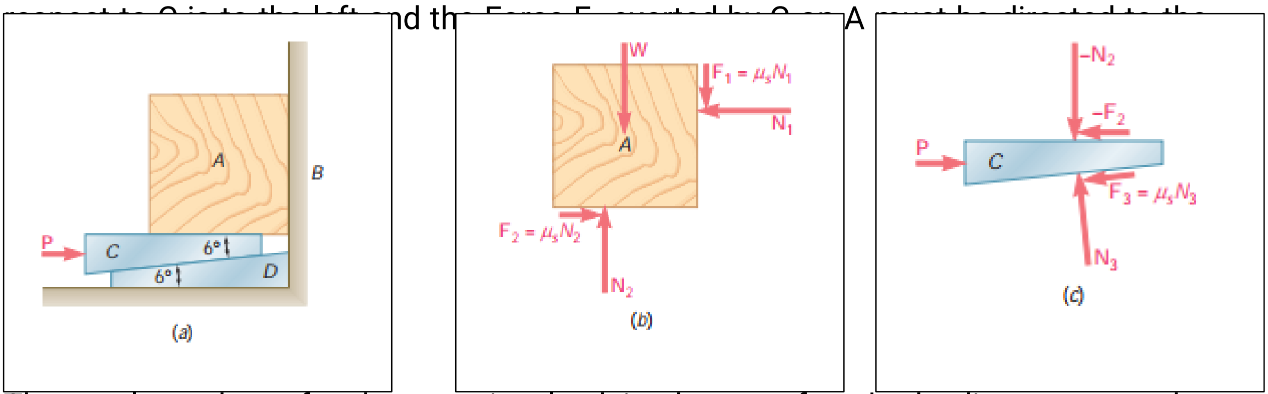
6. Determine the distance s to which the 90 kg painter can climb without causing the 4 m ladder to slip at the lower end A. the top of the 15 kg ladder has a small roller, and at the ground the coefficient of static friction is 0.25. The mass center of the painter is directly above her feet



Wedges

Wedges are simple machines used to raise large stone blocks and other heavy loads. These loads can be raised by applying to the wedge a force usually considerably smaller than the weight of the load. Also, due to friction between the surfaces in contact, a properly shaped wedge will remain in place after being forced under the load. Wedges can therefore be used advantageously to make small adjustments in the position of heavy pieces of machinery.

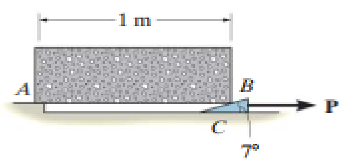
The block shown in figure (a) below rests on vertical wall B and is raised slightly by forcing a wedge C between block A and a second wedge D. To find the minimum value of P required to be applied to the wedge C to move the block. It will be assumed that the weight W of the block is known. The FBDs of block A and wedge C is shown in figure (b) and (c) respectively. The forces acting on the block include its weight and the normal and friction forces at the surfaces of contact with the wall B and wedge C. The magnitudes of the friction forces F_1 and F_2 are equal to $\mu_1 N_1$ and $\mu_2 N_2$ respectively since the motion of the block must be started. It is important to show the friction forces with the correct sense. Since the block will move upward, the force F_1 exerted by the wall on the block must be directed downward. On the other hand, since the wedge C moves to the right, the relative motion of A with respect to C is to the left and the Force F_2 exerted by C on A must be directed to the



The total number of unknowns involved in the two free body diagrams can be reduced to four if the friction forces are expressed in terms of the normal forces. Expressing that the block A and wedge C are in equilibrium will provide four equations which can be solved to obtain the magnitude of P

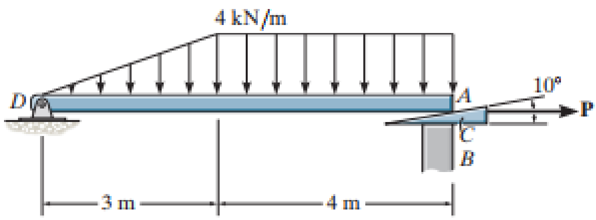
Example

The uniform stone shown in the figure below has a mass of 500 kg and is held in the horizontal position using a wedge at B. If the coefficient of static friction is $\mu_s = 0.3$ at the surfaces of contact, determine the minimum force P needed to remove the wedge. Assume that the stone dos not slip at A

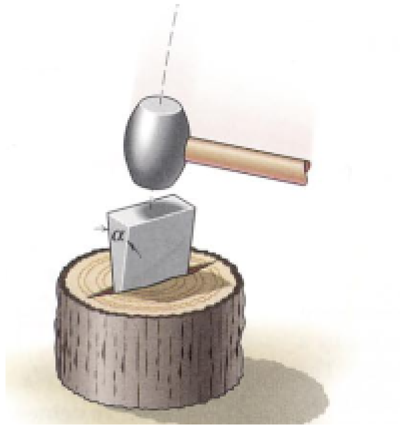


Practice Problems

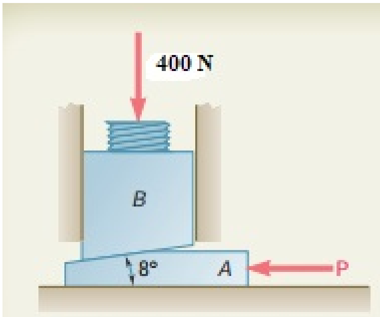
1. If bead AD is loaded as shown, determine the horizontal force P which must be applied to the wedge in order to remove it from under the beam. The coefficients of static friction at the top and bottoms surfaces of the wedge are $\mu_{CA} = 0.25$ and $\mu_{CB} = 0.35$, respectively. If P = 0, is the wedge self-locking? Neglect the weight and size of the wedge and the thickness of the beam.



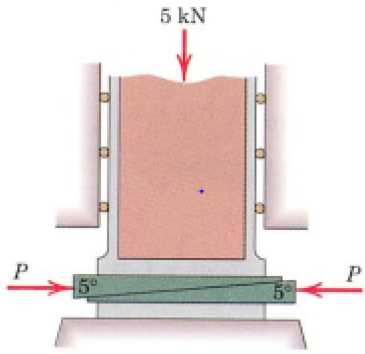
2. If the coefficient of friction between the steel wedge and the moist fibers of the newly cut stump is 0.20, determine the maximum angle α which the wedge may have and not pop out of the wood after being driven by the sledge



3. The position of the machine block B is adjusted by moving the wedge A. knowing that the coefficient of static fiction is 0.35 between all surfaces of contact, determine the force P required (a) to raise block B, (b) to lower block B



4. The two 5 wedges are shown are used to adjust the position of the column under a vertical load of 5 kN. Determine the magnitude of the forces P required to raise the column if the coefficient of friction for all surfaces is 0.40.



VIRTUAL WORK

Consider the imaginary movement of a body in static equilibrium, indicating a displacement or rotation that is assumed and does not actually exist. These movements are first order differential quantities and will be denoted by symbols δr and $\delta\theta$ respectively. The virtual work done by a force having a virtual displacement δr is

$\delta U = F\cos\theta\delta r$

Similarly, when a couple undergoes a virtual rotation $\delta\theta$ in the plane of the coupe forces, the virtual work is

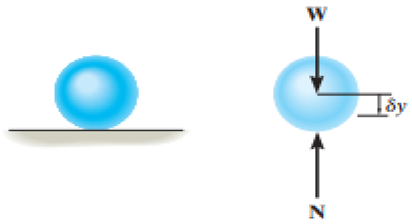
$\delta U = M\delta\theta$

Principle of Virtual Work

The principle of virtual work states that if a body is in equilibrium, then the algebraic sum of the virtual work done by all forces and couple moments acting on the body is zero for any virtual displacement of the body. Therefore

$\delta U = 0$

Consider the free body diagram of the particle (ball) that rest on the floor as shown in the figure below.

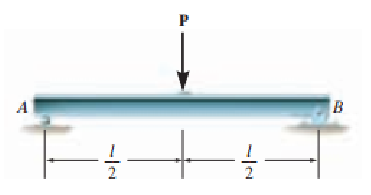


Assume the ball to be displaced downward by a virtual amount δy, then the weight does a positive virtual work Wδy and the normal force does negative virtual work, -Nδy. For equilibrium the total virtual work must be zero, so that

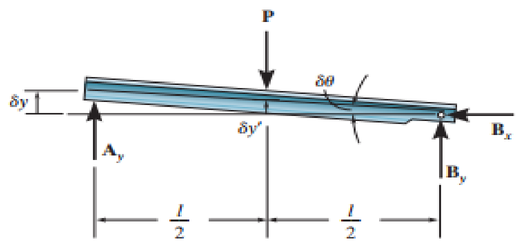
$$\delta U = W\delta y - N\delta y = (W-N)\delta y = 0$$

Since $\delta y \neq 0$, then N = W as required by applying $\sum F_y = 0$

For a simply supported beam as shown below, when the beam undergoes a virtual rotation δθ about point B, the only forces that do work are P and A_y. Since $\delta y = l d\theta$



(a)



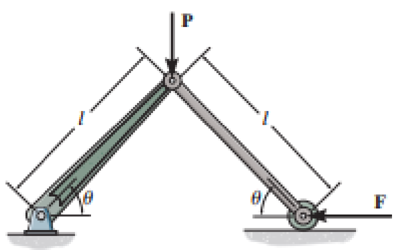
(b)

The virtual wok done is therefore

$$\delta U = A_y(l\delta\theta) - P\left(\frac{l}{2}\right)\delta\theta = 0 \qquad \text{Since } \delta\theta \neq 0\text{, then } A_y = \frac{P}{2}$$

Principle of Virtual Wok for a System of Connected Rigid Bodies

The method of virtual work is very effective for solving equilibrium problems that involve a system of several connected rigid bodies as shown in the figure below. The system has only one degree of freedom since the arrangement of the link can be completely specified using only one coordinate θ. We will restrict ourselves to only one degree of freedom in this course.

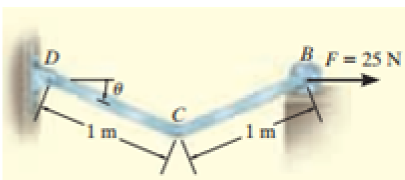


The procedures for solving problems involving a system of frictionless connected rigid bodies are highlighted below:

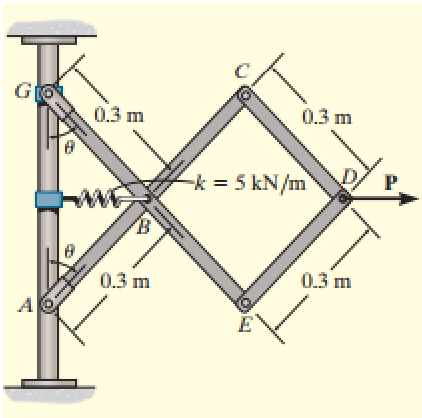
- Draw the FBD of the entire system of connected bodies and define the coordinate (q)
- Sketch the deflected position of the system on the FBD when the system undergoes a positive virtual displacement δq
- Indicate position coordinate s each measured from a fixed point on the FBD. These coordinates are directed to the forces that do work.
- Each of these coordinate axes should be parallel to the line of action of the force to which it is directed, so that the virtual work along the coordinate axis can be calculated.
- Relate each position coordinate s to the coordinate q; then differentiate these expressions in order to express each virtual displacement δs in terms of δq.
- Write the virtual work equation for the system assuming that each position coordinate s undergoes a positive virtual displacement δs. If a force or couple moment is in the same direction as the positive virtual displacement, the wok is positive, otherwise, it is negative
- Express the work in each force and coupe moments in the equation in terms of δq
- Factor out the common displacement from all the terms and solve the unknown force, couple moment, or equilibrium position q

Examples

1. Determine the angle θ for equilibrium of the two member linkage shown in the figure below. Each member has a mass of 10 kg

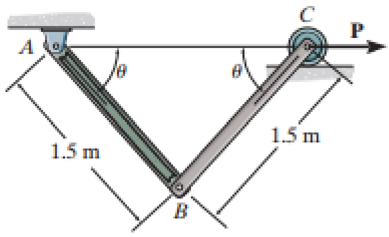


2. Determine the required force P in the figure below needed to maintain equilibrium of the scissors linkage when θ = 60°. The spring is unstretched when θ= 30°. Neglect the mass of the links

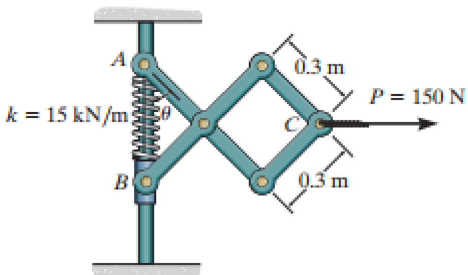


Practice Problems

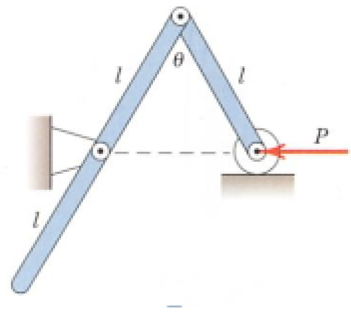
1. Determine the required magnitude of fore P to maintain equilibrium of the linkage at θ = 60°. Each link has a mass of 20 kg



2. The scissors linkage is subjected to a fore of P = 150 N. Determine the angle θ for equilibrium. The spring is unstretched at θ = 0°. Neglect the mass of the links



3. The mass of the uniform bar of length *l* is m while that if the uniform bar of length 2*l* is 2m. For a given force P, determine the angle θ for equilibrium



4. Determine the torque M on the activating lever of the dump truck necessary to balance the load of mass m with center of mass at G when the dump angle is θ. The polygon ABDC is a parallelogram