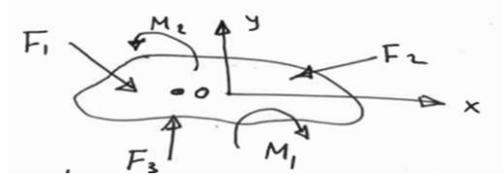


## EQUILIBRIUM

When a system of forces acting on body has no resultant, the body is said to be in equilibrium. Equilibrium means that the body remains in stability for both rest and moving states from Newton's law of motion. However, we will only consider static equilibrium. According to Newton's first law of motion, the equations of equilibrium for a rigid body in the plane as shown below are as follows

$$\begin{aligned} \sum F_x &= 0 \\ \sum F_y &= 0 \\ \sum M_o &= 0 \end{aligned}$$



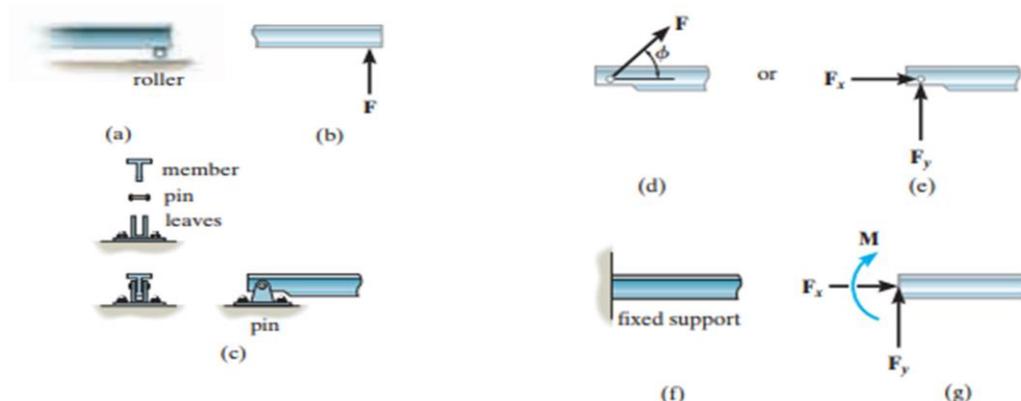
Where 'o' is any point in the plane

### Free Body Diagram (FBD)

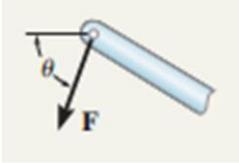
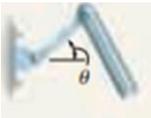
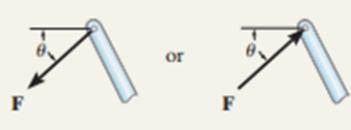
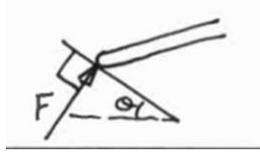
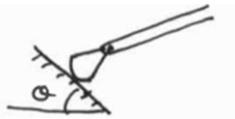
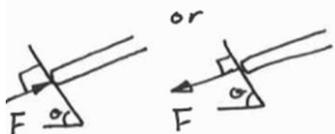
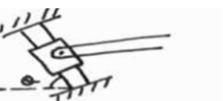
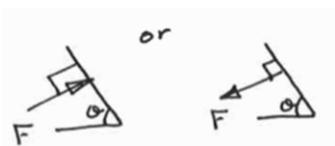
FBD is a sketch of a body or a portion of a body completely isolated (or free) from its surroundings. By employing the FBD, unknown forces or moments acting on a body can be determined through the equations of equilibrium. It is important to know three things before studying how to sketch the FBD

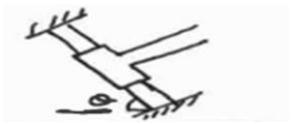
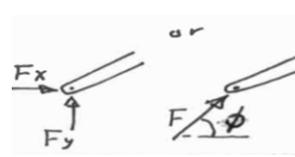
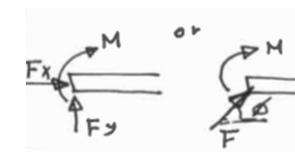
1. Support reactions: there are various types of reactions that occur at supports and points of support between bodies subjected to forces. There are various types of reactions that occur at supports and points of contact between bodies subjected to coplanar force systems. As a general rule:
  - A support prevents the translation of a body in a given direction by exerting a force on the body in the opposite direction.
  - A support prevents the rotation of a body in a given direction by exerting a couple moment on the body in the opposite direction

There are three main ways in which a horizontal member, such as a beam, is supported at its end. One method consists of a roller or cylinder, Figure (a). Since this support only prevents the beam from translating in the vertical direction, the roller will only exert a force on the beam in this direction as shown in Figure (b). The beam can be supported in a more restrictive manner by using a pin, shown in Figure C. The pin passes through a hole in the beam and two leaves which are fixed to the ground. Here the pin can prevent translation of the beam in any direction ( $\theta$ ) as shown in Figure d, and so the pin must exert a force  $F$  on the beam in the opposite direction. For purposes of analysis, it is generally easier to represent this resultant force  $F$  by its two rectangular components  $F_x$  and  $F_y$  as shown in Figure e. If  $F_x$  and  $F_y$  are known, then  $F$  and  $\theta$  can be calculated. The most restrictive way to support the beam would be to use a fixed support as shown in Figure f. This support will prevent both translation and rotation of the beam. To do this a force and couple moment must be developed on the beam at its point of connection, Figure g. As in the case of the pin, the force is usually represented by its rectangular components  $F_x$  and  $F_y$ .



Other common types of supports for bodies subjected to coplanar force systems is shown in the table below

S/No	Type of Connection	Reaction	Number of unknowns
1	 <p>Cable</p>		One (tension force)
2	 <p>Weightless link</p>		One (tension or compression) acting along the axis of the link
3	 <p>Roller</p>		One (reaction perpendicular to the surface at the point of contact)
4	 <p>Rocker</p>		One (reaction perpendicular to the surface at the point of contact)
5	 <p>Smooth contacting surface</p>		One (reaction perpendicular to the surface at the point of contact)
6	 <p>Roller on confined slot</p>		One (reaction perpendicular to the surface at the point of contact)
7	 <p>Member pin connected to a collar on a smooth rod</p>		One (reaction perpendicular to the surface at the point of contact)

8	 <p>Member fixed connected to a collar on a smooth rod</p>		Two (force and moment at the junction point)
9	 <p>Slider (or roller guide)</p>		Two (force and moment at the junction point)
10	 <p>Pin or hinge support</p>		Two (component of force or magnitude and direction of resultant force)
11	 <p>Fixed support</p>		Three (two components of force and moment or resultant force and moment)

## 2. External and Internal Forces

Since a rigid body is a composition of particles, both external and internal loadings may act on it. Only external loadings are represented on FBD because the net effect of internal forces on the body is zero

## 3. Weight of a Body

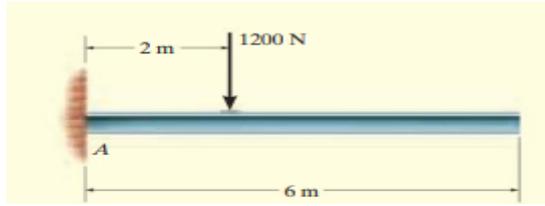
When a body is subjected to a gravitational field, then it has a specified weight. The weight of the body is represented by a resultant force directed downward and located at the center of gravity of the body. Note that that weight ( $w$ ) =  $mg$

### Procedure for Drawing the Free Body Diagram

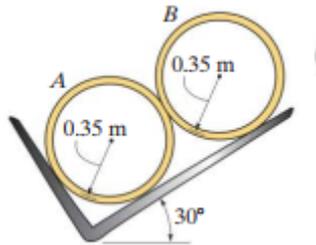
1. Isolate the body from its constraints and connections and draw its outlined shape
2. Identify all the external forces and moments that act on the body. These are generally due to
  - i. Applied loadings
  - ii. Reactions at the supports or at the points of contact with other bodies
  - iii. The weight of the body

### Examples

1. Draw the free body diagram and determine the reactions at the support of the uniform beam shown below. The beam has a mass of 50 kg

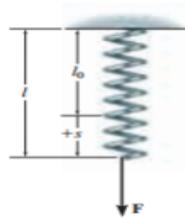


- Two smooth pipes each having a mass of 300 kg are supported by a fork tine of a tractor as shown below. Draw the FBD of each pipe and both pipes together



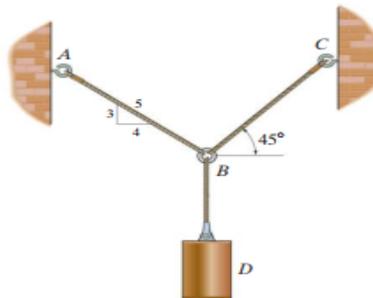
**Springs:** if we have a linearly elastic spring (linear means change in length proportional to the applied force and elastic means that the spring will return to its original shape after the applied force is removed). A characteristic that defines elasticity of a spring is called spring constant or stiffness ( $k$ ). The magnitude of force exerted on a linearly elastic spring having stiffness  $k$  and distance  $s = l - l_0$  is given by

$$F = ks$$

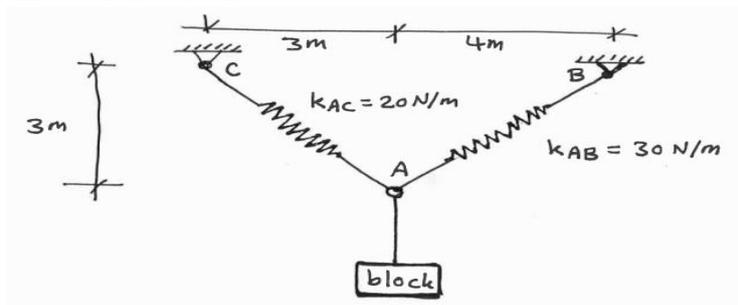


### Examples OF Equilibrium of coplanar Forces

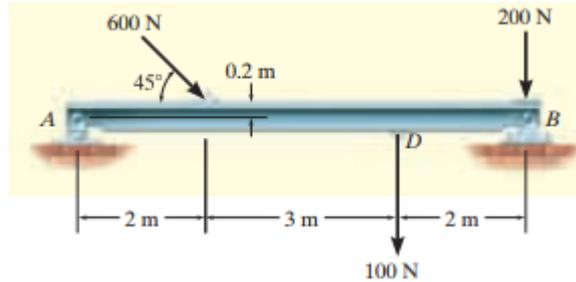
- Determine the tensions in cables BA and BC needed to support the 60 kg cylinder



- The unstretched length of spring AB is 3m. If the block is held in equilibrium position, determine the mass of the block.



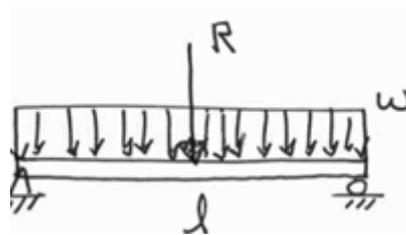
3. Determine the horizontal and vertical components of reaction on the beam caused by the pin at  $B$  and the rocker at  $A$  as shown in Figure below. Neglect the weight of the beam.



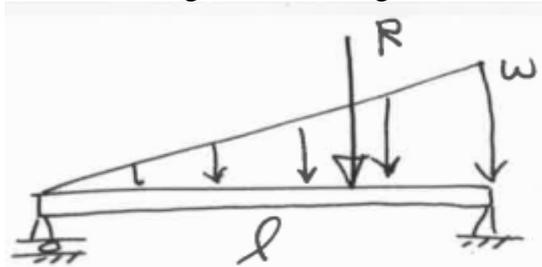
### Resultant of Distributed Loading

A body may be subjected to distributed loadings such as those caused by wind, fluid, or weight of material over the surface of the body. The most common types of these loads are the uniform loading and triangular loading

- i. Uniformly distributed loading: the magnitude of the resultant is  $R = w \times l$  or (area of the load). The location of  $R$  passes through the centroid of the rectangle (geometric center/middle)

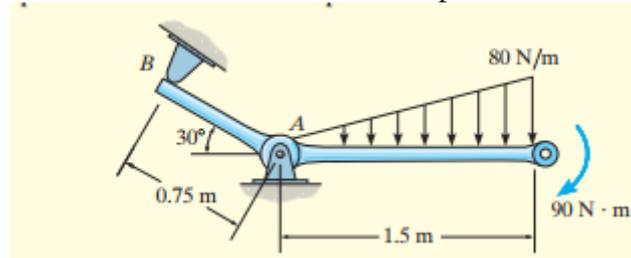


- ii. Triangular loading: the resultant  $R$  is the area of triangle, or  $R = \frac{wl}{2}$  at the centroid of the triangle (one-third of the length of the triangle measured from the base)



### Example

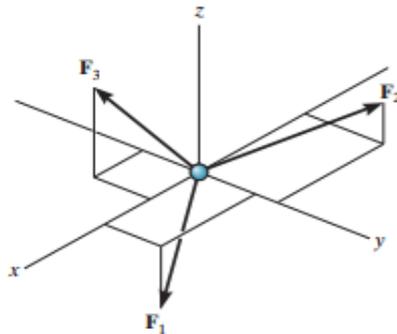
The member shown in the figure below is pin connected at  $A$  and rests against a smooth support at  $B$ . Determine the horizontal and vertical components of reaction at the pin  $A$ .



## Equilibrium in Three Dimensional Force System

Remembering that the necessary and sufficient condition for equilibrium of a particle is

$$\sum F = 0$$



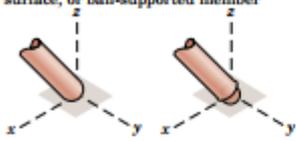
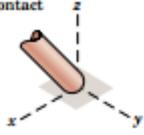
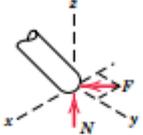
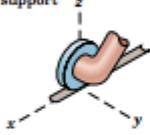
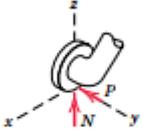
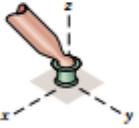
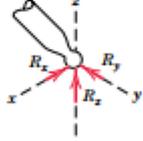
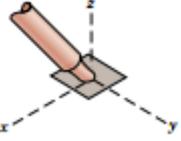
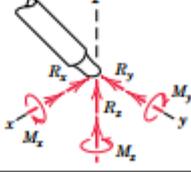
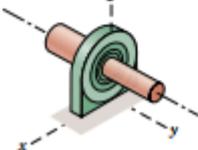
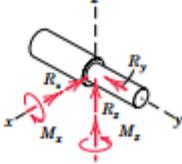
For a three dimensional force system as shown in in the figure above, the forces can be resolved into their respective  $i$ ,  $j$ ,  $k$  components such that  $\sum F_x i + \sum F_y j + \sum F_z k = 0$  to satisfy this equation, it is required that

$$\sum F_x i = 0$$

$$\sum F_y j = 0$$

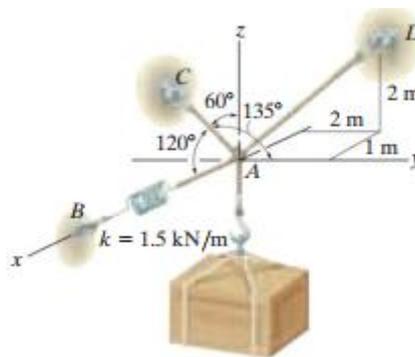
$$\sum F_z k = 0$$

These three equations states that the sum of the components of all forces acting on the particle along each coordinates axes must be zero. Using these equations we can solve for t most three unknowns which are represented as coordinate direction angles or magnitudes of forces as shown on the particle's free body diagram

MODELING THE ACTION OF FORCES IN THREE-DIMENSIONAL ANALYSIS	
Type of Contact and Force Origin	Action on Body to Be Isolated
<p>1. Member in contact with smooth surface, or ball-supported member</p> 	 <p>Force must be normal to the surface and directed toward the member.</p>
<p>2. Member in contact with rough surface</p> 	 <p>The possibility exists for a force <math>F</math> tangent to the surface (friction force) to act on the member, as well as a normal force <math>N</math>.</p>
<p>3. Roller or wheel support with lateral constraint</p> 	 <p>A lateral force <math>P</math> exerted by the guide on the wheel can exist, in addition to the normal force <math>N</math>.</p>
<p>4. Ball-and-socket joint</p> 	 <p>A ball-and-socket joint free to pivot about the center of the ball can support a force <math>R</math> with all three components.</p>
<p>5. Fixed connection (embedded or welded)</p> 	 <p>In addition to three components of force, a fixed connection can support a couple <math>M</math> represented by its three components.</p>
<p>6. Thrust-bearing support</p> 	 <p>Thrust bearing is capable of supporting axial force <math>R_y</math> as well as radial forces <math>R_x</math> and <math>R_z</math>. Couples <math>M_x</math> and <math>M_z</math> must, in some cases, be assumed zero in order to provide statical determinacy.</p>

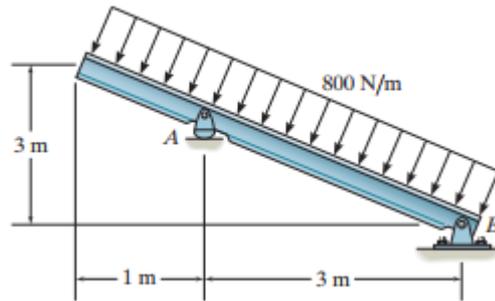
### Example

- Determine the force in each cord used to support the 100kg crate shown below

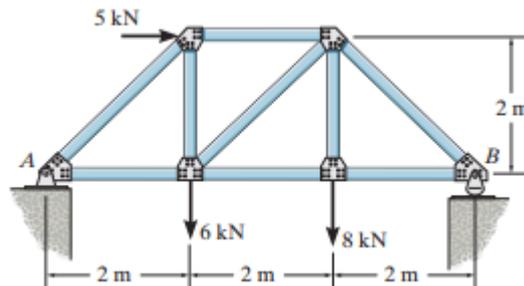


### Practice Problems

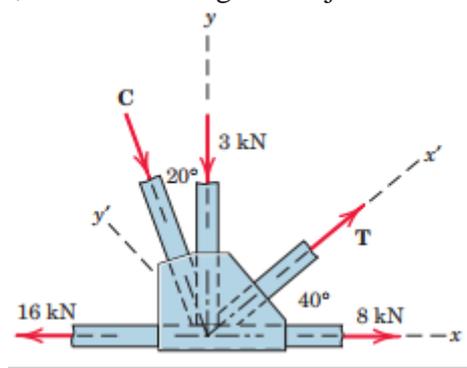
1. Determine the reactions at the support



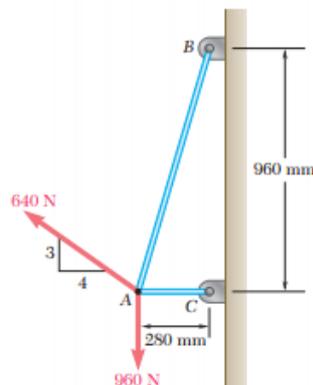
2. Determine the reactions at the support



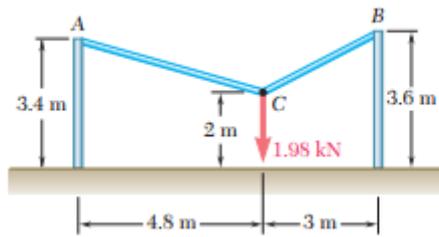
3. Determine the magnitudes of the forces  $C$  and  $T$ , which, along with the other three forces shown, act on the bridge-truss joint



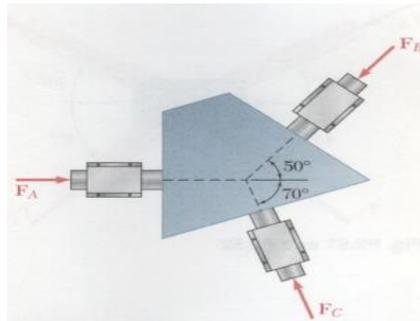
4. Two cables are tied together at  $A$  and loaded as shown. Draw the free-body diagram needed to determine the tension in each cable.



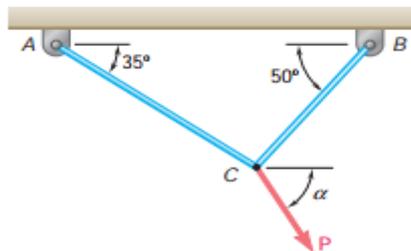
5. Two cables are tied together at  $C$  and loaded as shown. Determine the tension (a) in cable  $AC$ , (b) in cable  $BC$



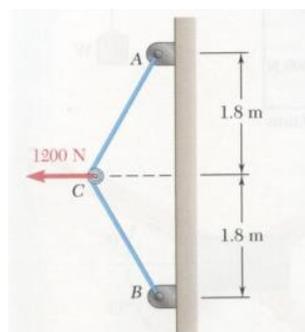
6. An irregularly shaped machine component is held in the position by three clamps. Knowing that  $F_A = 940$  N, determine the magnitudes of the forces  $F_B$  and  $F_C$  exerted by the other two clamps



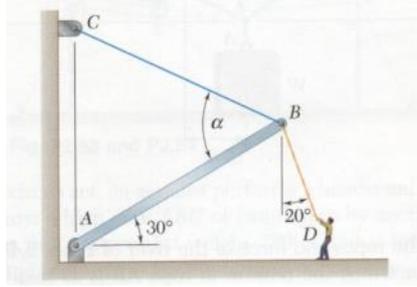
7. Two cables tied together at  $C$  are loaded as shown. Knowing that the maximum allowable tension in each cable is 800 N, determine (a) the magnitude of the largest force  $P$  that can be applied at  $C$ , (b) the corresponding value of  $\alpha$



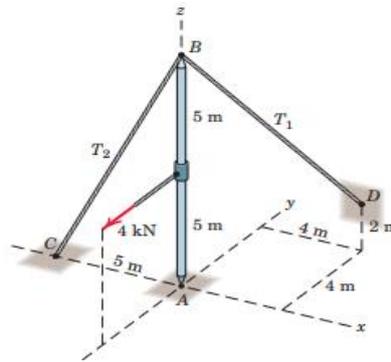
8. Knowing that portions  $AC$  and  $BC$  of cable  $ACB$  must be equal, determine the shortest length of cable that can be used to support the load shown if the tension in the cable is not to exceed 870 N.



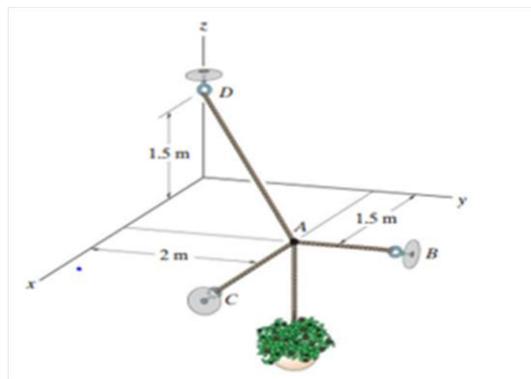
9. Boom  $AB$  is supported by cable  $BC$  and a hinge at  $A$ . Knowing that the boom exerts on pin  $B$  a force directed along the boom and that the tension in rope  $BD$  is 310 N, determine (a) the value of  $\alpha$  for which the tension in cable  $BC$  is as small as possible, (b) the corresponding value of the tension.



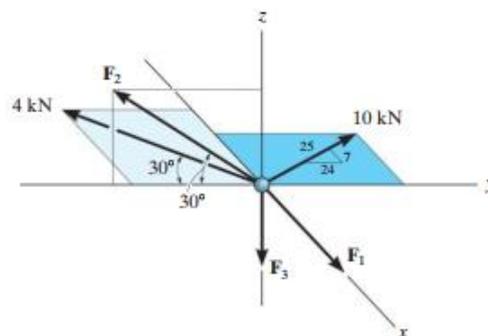
10. The vertical mast supports the 4-kN force and is constrained by the two fixed cables  $BC$  and  $BD$  and by a ball-and-socket connection at  $A$ . Calculate the tension in  $BD$ . Can this be accomplished by using only one equation of equilibrium?



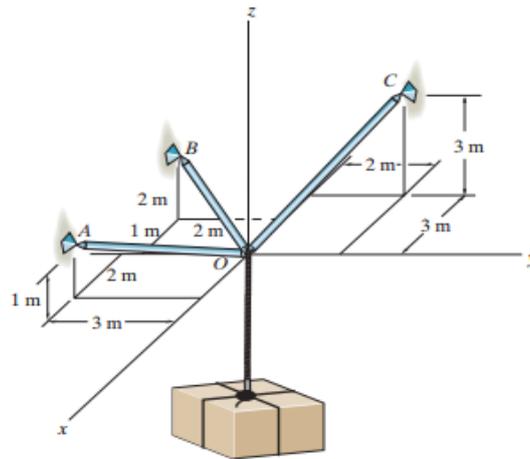
11. The three cables are used to support the 40 kg flowerpot. Determine the force developed in each cable for equilibrium



12. Determine the magnitude of  $F_1$ ,  $F_2$  and  $F_3$  for equilibrium of the particle



13. If the maximum force in each rod cannot exceed 1500 N, determine the greatest mass of the crate that can be supported



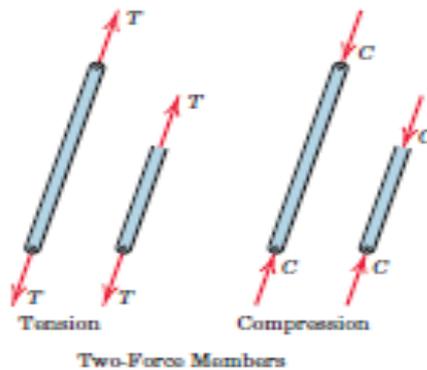
## ANALYSIS OF STRUCTURES

A structure is an assemblage of a number of resistant bodies. There are three broad categories of engineering structures

1. Trusses: a truss is a structure composed of slender members joined together at their end points. They are designed to carry loads and they are usually stationary fully constrained structures. Trusses consist exclusively of straight members connected at the joints located at the end of each member. Members of a truss are therefore two force members, that is, members acted upon by two equal and opposite forces directed along the members.
2. Frames: they are also designed to support loads and are usually stationary fully constrained structures. However, they contain at least one multiforce member, that is, a member acted upon by three or more forces which in general are not directed along the member.
3. Machines: they are designed to transmit and modify forces. They are structures containing moving parts. Machines, like frames, always contain at least one multiforce member.

### Analysis of Simple Trusses

In a truss, all loadings are applied at the joint, the forces at the end of the member are directed along the axis of the members and this force tends to elongate the member (Tensile force, T) or shorten the member (Compressive force, C)



Two methods are available employed in the analysis of a truss

- Method of joints
- Method of section

### Method of Joint

In order to analyze or design a truss, it is necessary to determine the force in each of its members. One way to do this is to use the method of joints. This method is based on the fact that if the entire truss is in equilibrium, then each of its joints is also in equilibrium. Therefore, if the free-body diagram of each joint is drawn, the force equilibrium equations can then be used to obtain the member forces acting on each joint. Since the members of a plane truss are straight two-force members lying in a single plane, each joint is subjected to a force system that is coplanar and concurrent. As a result, only  $\sum F_x = 0$  and  $\sum F_y = 0$  need to be satisfied for equilibrium.

When using the method of joints, always start at a joint having at least one known force and at most two unknown forces. In this way, application of  $\sum F_x = 0$  and  $\sum F_y = 0$  yields two algebraic equations which can be solved for the two unknowns. When applying these equations, the correct sense of an unknown member force can be determined by inspection or by assuming all unknown forces acting on a joint's FBD are in tension (i.e, pulling on the

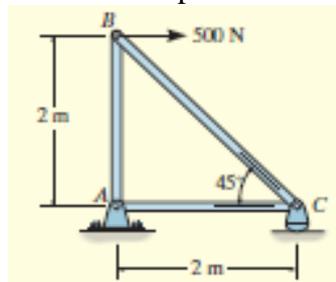
pin). If this is done, then numerical solution of the equilibrium equations will yield positive scalars for members in tension and negative scalars for members in compression. Once an unknown member force is found, use its correct magnitude and sense (T or C) on subsequent joint free-body diagrams.

### Procedure for Analysis

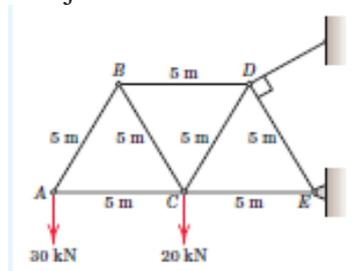
1. Draw the FBD of a joint having at least one unknown force and at most two unknown member forces
  2. If the sense of direction of the force is unknown, assume the force is tension
  3. Apply  $\sum F_x = 0$  and  $\sum F_y = 0$  to find the unknown member forces
  4. Repeat steps (1) to (3) for other joints.
- Note: orient x and y axes such that the forces in F.B.D can be easily determined

### Example

- i. Determine the force in each member of the truss shown in the figure below and indicate whether the members are in tension or compression



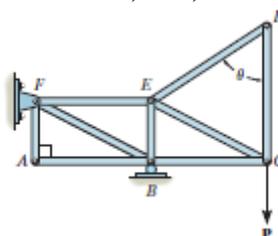
- ii. Determine the external forces and compute the force in each member of the loaded cantilever truss by the method of joints.



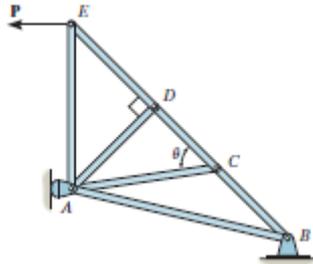
### Zero-Force Members

Truss analysis using the method of joints is greatly simplified if we can first identify those members which support no loading. These members are known as zero-force members and are used to increase the stability of the truss during construction and to provide added support if the loading is changed. Zero-force members of a truss can generally be found by inspection of each of the joints and there are two general cases.

- i. if only two non-collinear members form a truss joint and no external load or support reaction is applied to the joint, the two members must be zero-force members. For example, in the figure below members AF, AB, DE and DC are all zero force members

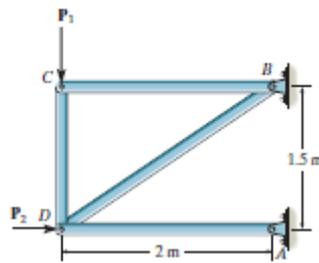


- ii. If three members form a truss joint for which two of the members are collinear, the third member is a zero-force member provided no external force or support reaction has a component that acts along this member. For example, members DA and CA as shown in the figure below

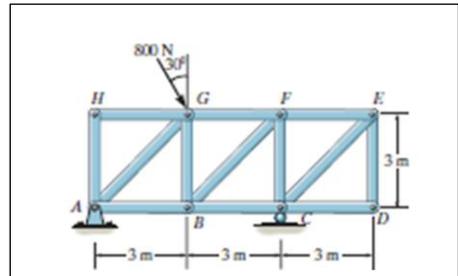
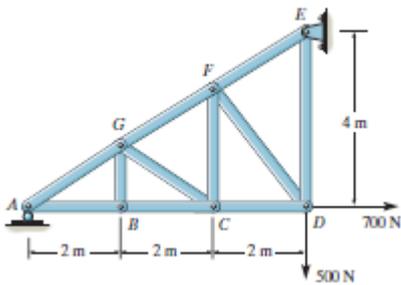


**Practice Problems**

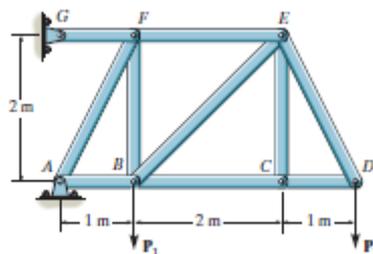
1. Determine the force in each member of the truss and state if the members are in tension or compression. Set  $P_1 = 45 \text{ kN}$ ,  $P_2 = 30 \text{ kN}$ .



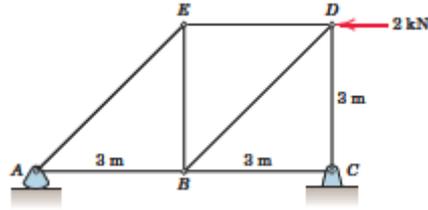
2. Identify the zero force members in the figures below



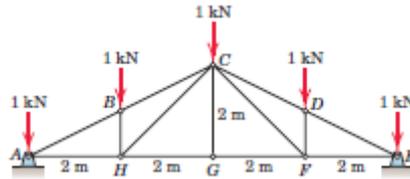
3. Determine the force in each member of the truss and state if the members are in tension or compression. Set  $P_1 = 8 \text{ kN}$ ,  $P_2 = 12 \text{ kN}$



4. Calculate the force in each member of the loaded truss.

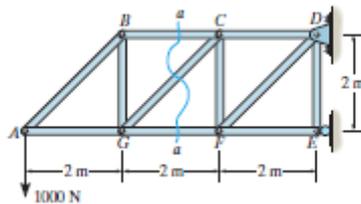


5. A snow load transfers the forces shown to the upper joints of a Pratt roof truss. Neglect any horizontal reactions at the supports and solve for the forces in all members.



### Method of Section

Sometimes, we need to find the force in only a few members of a truss, we can analyze the truss using the method of sections. It is based on the principle that if the truss is in equilibrium then any segment of the truss is also in equilibrium. For example, consider the two truss members shown on the left in the figure below. If the forces within the members are to be determined, then an imaginary section, indicated by the wavy line, can be used to cut each member into two parts and thereby “expose” each internal force as “external” to the free-body diagrams shown. The equation of equilibrium is then applied to the isolated part of the truss.

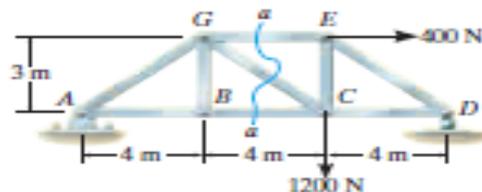


### Procedure for analysis using method of section

- Select a section that passes through the members with the unknown forces
- Draw the FBD of the part of the truss with the lowest number of forces acting on it
- Apply the equations of equilibrium to find the unknown.

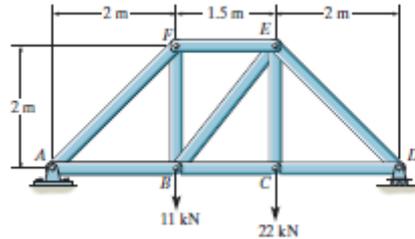
### Example

Determine the force in members GE, GC, and BC of the truss shown the figure below. Indicate whether the members are in tension or compression.

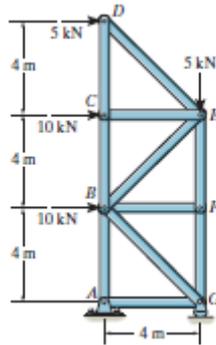


### Practice Problems

- Determine the force developed in members FE, EB, and BC of the truss and state if these members are in tension or compression.



2. Determine the force in members BF, BG, and AB, and state if the members are in tension or compression.



### Frames and Machines

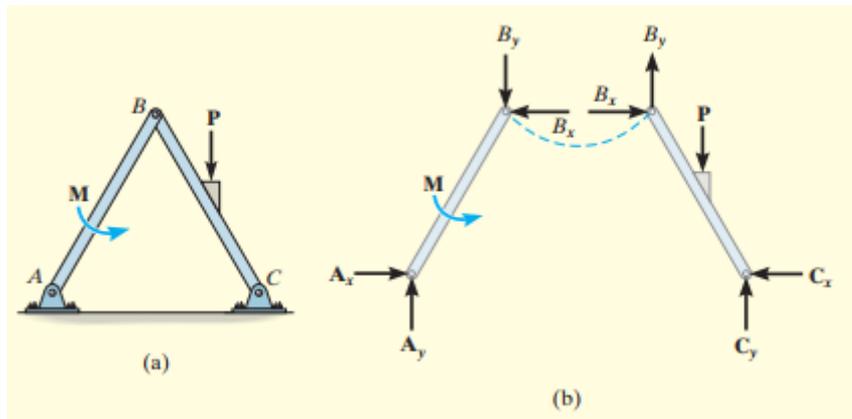
A structure is called a frame or machine if at least one of its individual members is a multiforce member. A multiforce member is defined as one with three or more forces acting on it, or one with two or more forces and one or more couples acting on it. Frames and machines are two common types of structure which are often composed of pin connected parts. Frames are used to support loads whereas machines which contain moving parts are used to transmit and alter forces. In the analysis of frames and machines, the FBD for the parts are drawn, and the three equations of equilibrium can be applied for the parts (and/or) for the entire structure. When the FBD is drawn for the pin connected parts, the internal reactions at the connection pin must be equal and opposite to each other.

### Free-Body Diagrams

In order to determine the forces acting at the joints and supports of a frame or machine, the structure must be disassembled and the free-body diagrams of its parts must be drawn. The following important points *must* be observed

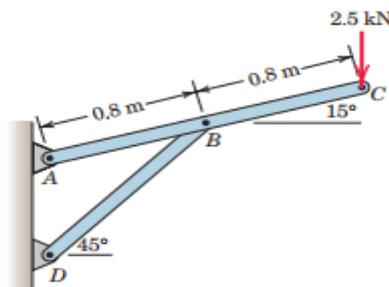
- i. Isolate each part by drawing its *outlined shape*. Then show all the forces and/or couple moments that act on the part. Make sure to *label* or *identify* each known and unknown force and couple moment with reference to an established  $x, y$  coordinate system. Also, indicate any dimensions used for taking moments. Most often the equations of equilibrium are easier to apply if the forces are represented by their rectangular components.
- ii. Identify all the two-force members in the structure and represent their free-body diagrams as having two equal but opposite collinear forces acting at their points of application. By recognizing the two-force members, we can avoid solving an unnecessary number of equilibrium equations.
- iii. Forces common to *any* two *contacting* members act with equal magnitudes but opposite sense on the respective members. If the two members are treated as a “*system*” of *connected members*, then these forces are “*internal*” and are *not shown* on the *free-body diagram of the system*; however, if the free-body diagram of *each member* is drawn, the forces are “*external*” and *must* be shown as

equal in magnitude and opposite in direction on each of the two free-body diagrams as shown below

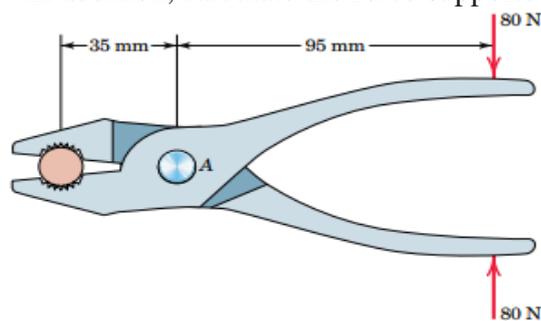


## Examples

1. Determine the force in member  $BD$  and the forces supported by all pins in the loaded frame.

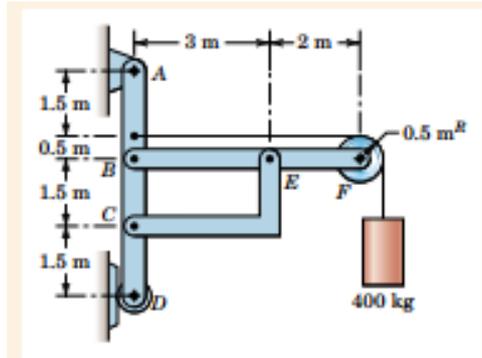


2. For an 80-N squeeze on the handles of the pliers, determine the force  $F$  applied to the round rod by each jaw. In addition, calculate the force supported by the pin at  $A$ .

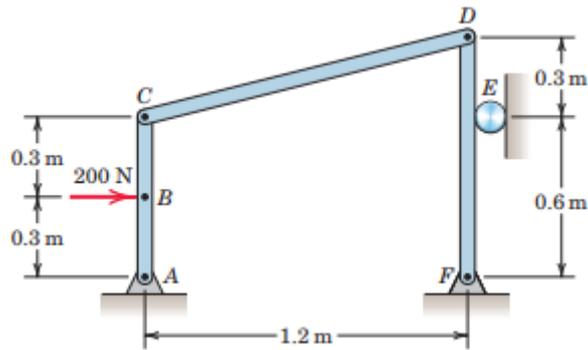


## Practice Problems

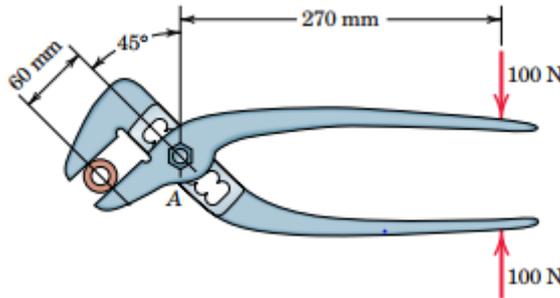
1. The frame supports the 400-kg load in the manner shown. Neglect the weights of the members compared with the forces induced by the load and compute the horizontal and vertical components of all forces acting on each of the members.



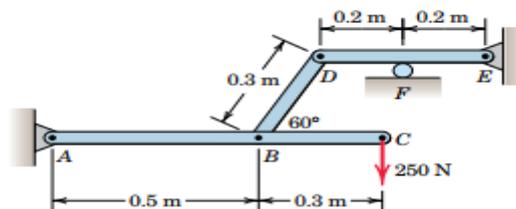
2. Determine the force supported by the roller at  $E$



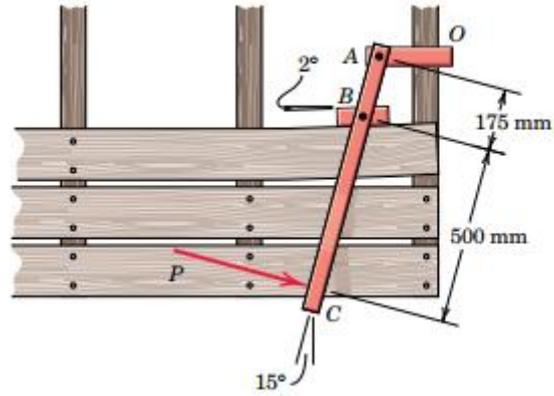
3. Compute the force supported by the pin at  $A$  for the slip-joint pliers under a grip of 100 N



4. Determine the reaction at the roller  $F$  for the frame loaded as shown.



5. The device shown is used to straighten bowed decking boards just prior to final nailing to the joists. There is a lower bracket (not shown) at  $O$  which fixes the part  $OA$  to a joist, so that the pivot  $A$  may be considered fixed. For a given force  $P$  exerted perpendicular to the handle  $ABC$  as shown, determine the corresponding normal force  $N$  applied to the bent board near point  $B$ . Neglect friction.



6. Determine the moment  $M$  which must be applied at A to keep the frame in static equilibrium in the position shown. Also calculate the magnitude of the pin reaction at A

