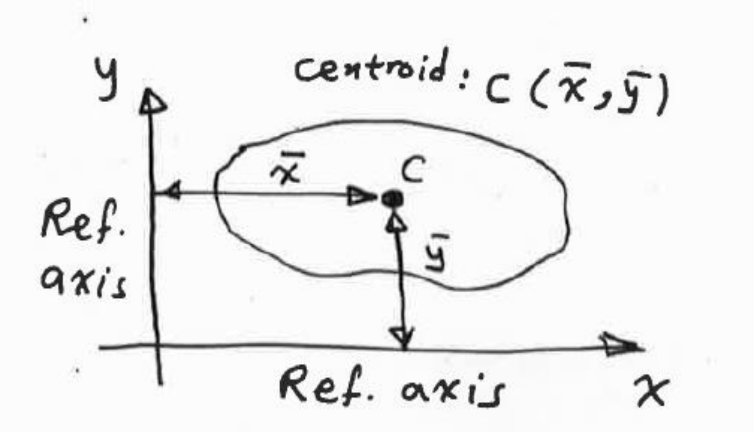
CENTROID

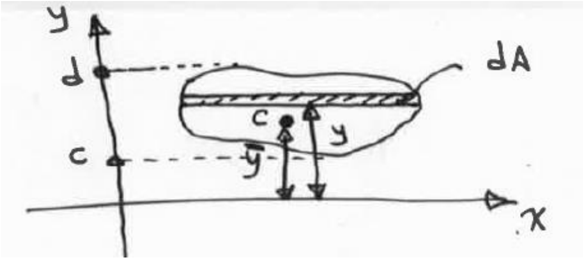
Centroid is the point which defines the geometric center of an object. Lines, areas and volume all have centroids we will however be considering the centroids of areas. Centroid of an area lies on the axis of symmetry if it exits. Centre of gravity is applied to bodies with mass and weight and centroid is applied to plane areas.

Centroids of Areas

The centroid of area is represented by the coordinates $(\bar{x}, \bar{y})$ measured from a reference axes x and y



Generally, the centroid of an area is determined as follows



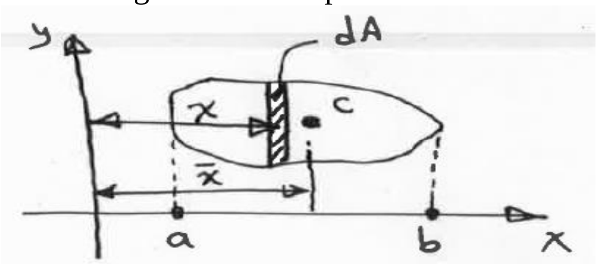
Take a horizontal strip as shown above

Moment of area $= \sum$ Moments of strips

$A.\bar{y} = \int dA.y$

$\bar{y} = \frac{\int dA.y}{A}$

Also taking a vertical strip as shown below



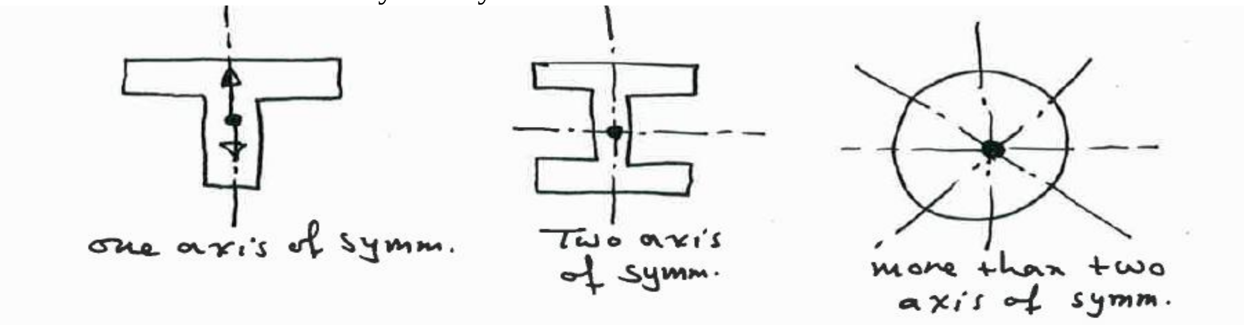
Moment of area $= \sum$ Moments of strips

$A.\bar{x} = \int dA.x$

$\bar{x} = \frac{\int dA.x}{A}$

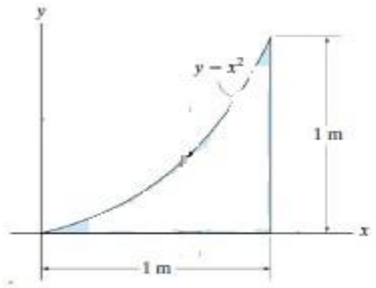
Symmetric Areas

When an area has an axis of symmetry, the centroid of the area will lie on the axis of the symmetry. When the area has more than one axes of symmetry, the centroid will lie at the intersection of these axes of symmetry.

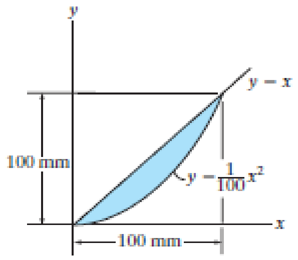


Example

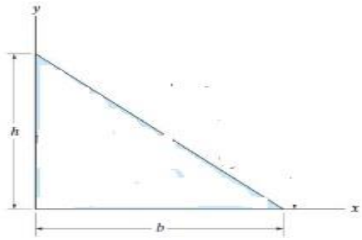
1. Locate the centroid of the area under the curve shown below



2. Locate the centroid y of the shaded area.



3. Determine the distance y measured from the x axis to the centroid of the area of the triangle shown in the figure below



Centroid of Composite Area

The centroid of composite area can be found using the relations

$\bar{x} = \frac{\sum xA}{\sum A}$ and $\bar{y} = \frac{\sum yA}{\sum A}$

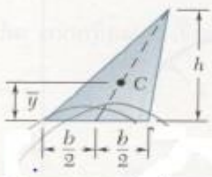
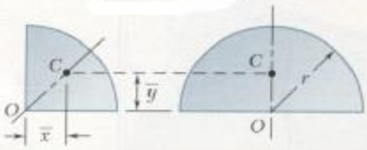
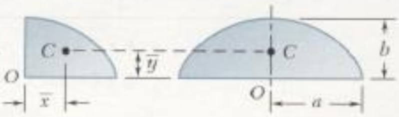
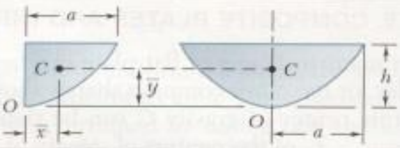
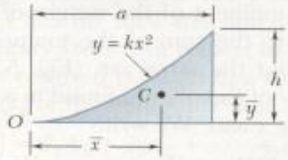
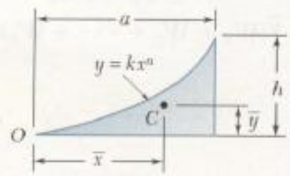
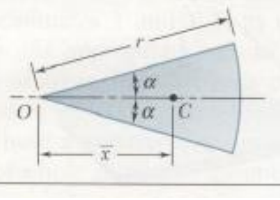
Where

x,y = centroids of each composite part of the area

$\sum A$ = sum of the areas of all parts (total area)

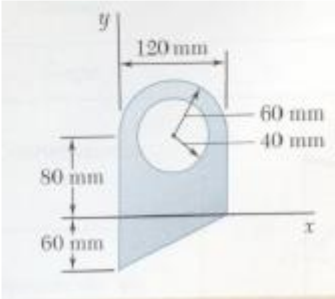
$\bar{x}, \bar{y}$ = centroid of the total area

Centroids of Some Common Shapes

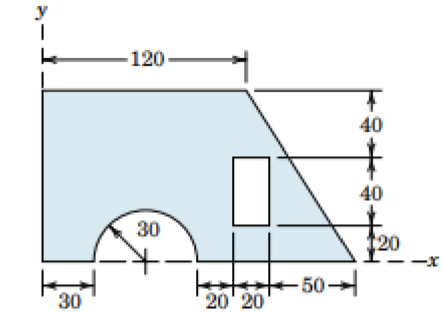
Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
General spandrel		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Circular sector		$\frac{2r \sin\alpha}{3\alpha}$	0	αr^2

Examples

1. For the plane area shown determine the location of the centroid

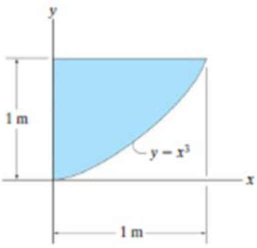


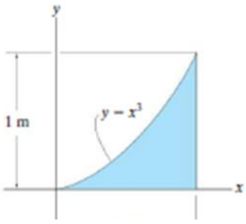
2. Determine the *x*- and *y*-coordinates of the centroid of the shaded area.



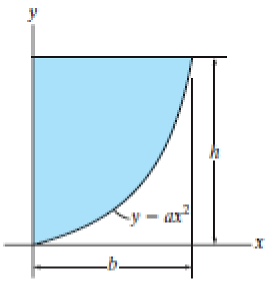
Practice Problems

1. Determine the centroid $(\bar{x}, \bar{y})$ of the shaded area.

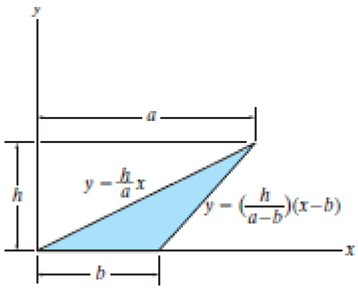




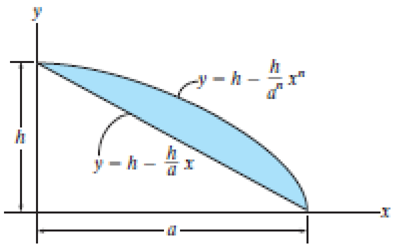
2. Locate the centroid $\bar{x}$ of the parabolic area shown below



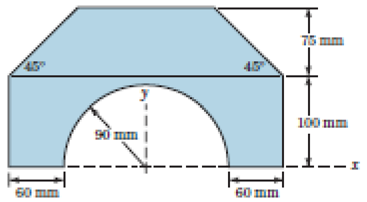
3. Determine the centroid $(\bar{x}, \bar{y})$ of the shaded area



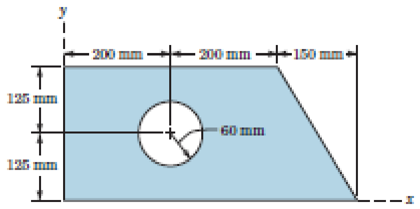
4. Determine the centroid $(\bar{x}, \bar{y})$ of the shaded area



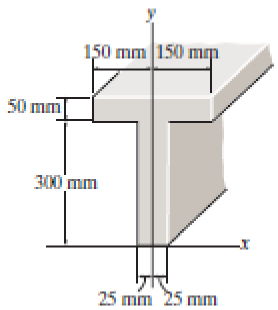
5. Determine the *y*-coordinate of the centroid of the shaded area.



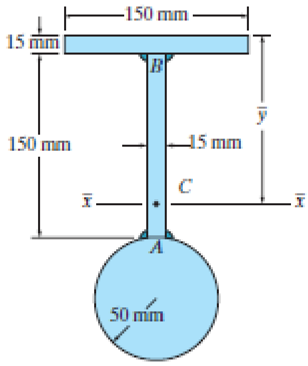
6. Determine the coordinates of the centroid of the shaded area.



7. Locate the centroid *y* of the beam's cross-sectional area.



8. Determine the location y of the centroidal axis x-x of the beam's cross-sectional area. Neglect the size of the corner welds at A and B for the calculation.

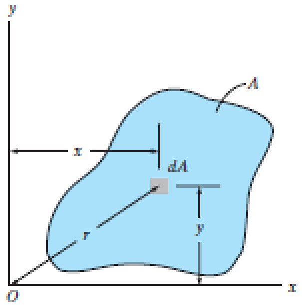


150 mm
15 mm
B
150 mm
15 mm
$\bar{y}$
C
$\bar{x}$
$\bar{x}$
A
50 mm

MOMENT OF INERTIA

Inertia is the resistance of any object to changes. The moment of inertia of an object is a measure of its resistance to changes in its rotation. Moment of inertia for an area is very important in the design and analysis of structural members. Centroid of a body represents its first moment of area ($\int xdA$). While the moment of inertia of area represents the second moment of area ($\int x^2dA$).

For the figure shown below the moment of inertia with respect to the x and y axes are



$I_x = \int y^2dA$

$I_y = \int x^2dA$

Moment of inertia of the small area dA can also be formulated about the pole 'O' or z axis as

$J_o = \int r^2dA$

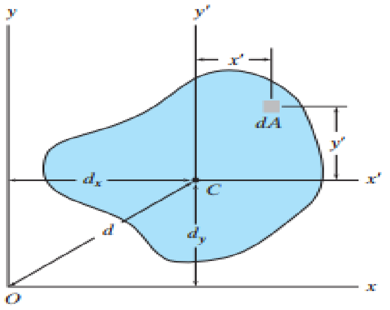
A relationship can be established between $I_x, I_y and J_o$ since $r^2 = x^2 + y^2$. Therefore,

$J_o = \int r^2dA = \int (x^2 + y^2)dA = I_x + I_y$

Moment of inertia is always positive since it involves the product of the square of the distance and area. The unit of moment of inertia involves the length raised to the fourth power for example mm^4, cm^4, m^4

Parallel Axis Theorem

The ***parallel-axis theorem*** can be used to find the moment of inertia of an area about **any axis** that is parallel to an axis passing through the centroid and about which the moment of inertia is known. To develop this theorem, consider the moment of inertia of the shaded area shown in the figure below about the **x** axis. To start, we choose a differential element **dA** located at an arbitrary distance y**'** from the **centroidal** x' axis. If the distance between the parallel **x** and x' axis is dy, then the moment of inertia of **dA** about the **x** axis is



$dI_x = (y' + dy)^2\ dA$. For the entire area,

$$I_x = \int (y' + dy)^2 \, dA$$
$$= \int y'^2 \, dA + 2d_y \int y' dA + d_y^2 \int dA$$

The first integral represents the moment of inertia of the area about the centroidal axis ($\bar{I}_x$). The second integral is zero since x'axis passes through the area's centroid C, that is, $\int y'dA = \bar{y}' \int dA = 0$. Since $\bar{y}' = 0$. The third integral represents the total area (A), the final result therefore is

$$I_x = \bar{I}_x + Ad_y^2$$

The expression can also be written for y as

$$I_y = \bar{I}_y + Ad_x^2$$

And also for the polar moment, knowing that $\bar{J}_c = \bar{I}_x + \bar{I}_y$ and $d = d_x^2 + d_y^2$

$$J_o = \bar{J}_c + Ad^2$$

The form of each of these three equations states that the moment of inertia for an area about an axis is equal to its moment of inertia about a parallel axis passing through the area's centroid plus the product of the area and the square of the perpendicular distance between the axes.

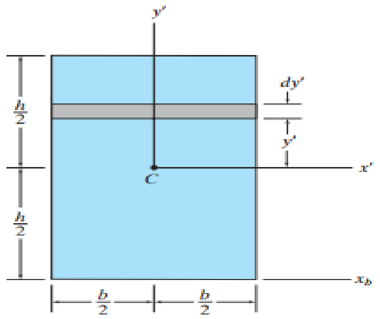
Radius of Gyration

The ***radius of gyration*** of an area about an axis has units of length and is a quantity that is often used for the design of columns in structural mechanics. Provided the areas and moments of inertia are **known**, the radii of gyration are determined from the formulas

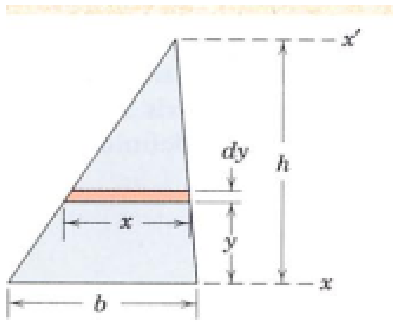
$$k_x = \sqrt{\frac{I_x}{A}}$$
$$k_y = \sqrt{\frac{I_y}{A}}$$
$$k_o = \sqrt{\frac{J_o}{A}}$$

Examples

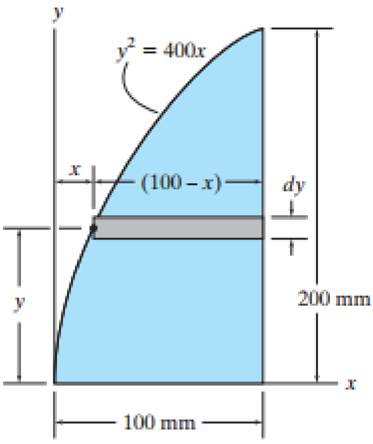
i. Determine the moment of inertia for the rectangular area shown in the figure below with respect to (a) the centroidal x' axis, (b) the axis x_b passing through the base of the rectangle, and (c) the pole or z' axis perpendicular to the x'-y' plane and passing through the centroid C



ii. Determine the moments of inertia of the triangular area shown below about its base and about parallel axes through its centroid and the vertex



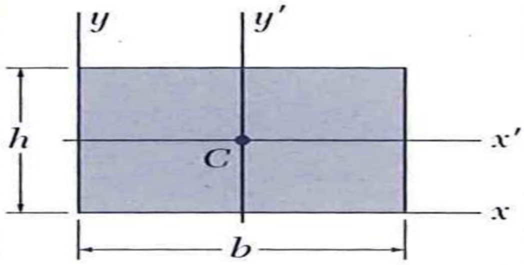
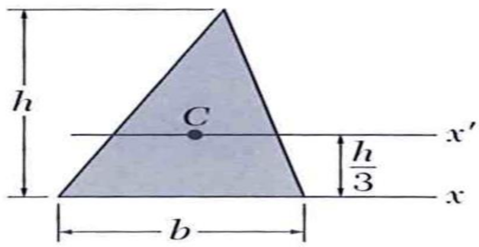
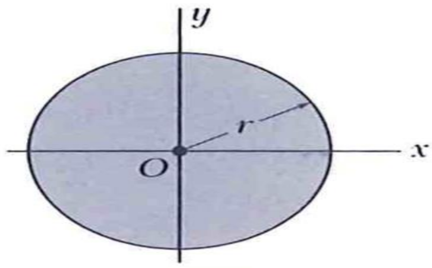
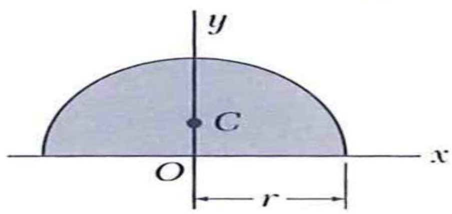
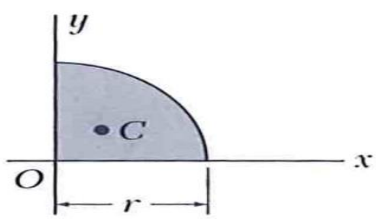
iii. Determine the moment of inertia of the shaded area about the x axis .



Moment of Inertia for Composite Areas

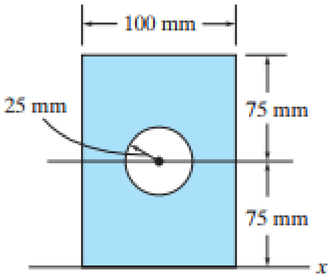
A composite area consists of a series of connected “simpler” parts or shapes, such as rectangles, triangles, and circles. Provided the moment of inertia of each of these parts is known or can be determined about a common axis, then the moment of inertia for the composite area about this axis equals the algebraic sum of the moments of inertia of all its parts

Moments of Inertia of Common Geometric Shapes

Rectangle $\bar{I}_{x'} = \frac{1}{12}bh^3$ $\bar{I}_{y'} = \frac{1}{12}b^3h$ $I_x = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}b^3h$ $J_C = \frac{1}{12}bh(b^2 + h^2)$	
Triangle $\bar{I}_{x'} = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$	
Circle $\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$	
Semicircle $I_x = I_y = \frac{1}{8}\pi r^4$ $J_O = \frac{1}{4}\pi r^4$	
Quarter circle $I_x = I_y = \frac{1}{16}\pi r^4$ $J_O = \frac{1}{8}\pi r^4$	

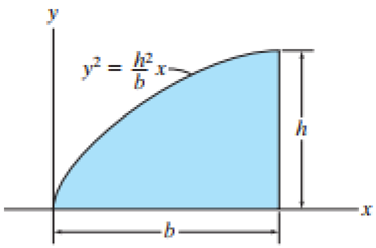
Example

i. Determine the moment of inertia of the area shown in the figure below about the x axis

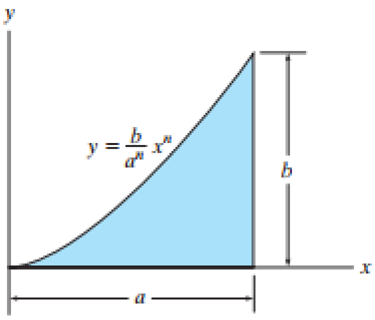


Practice Problems

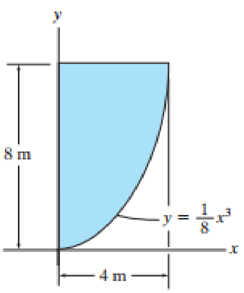
i. Determine the moment of inertia of the area about the *x* axis.



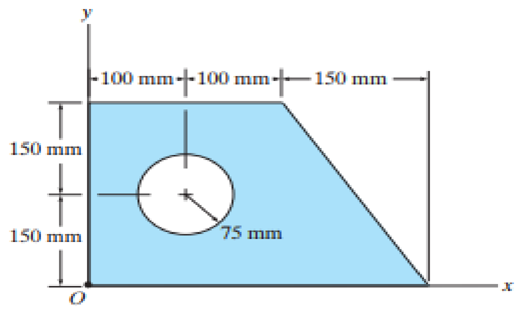
ii. Determine the moment of inertia of the shaded area about the x and y axes



iii. Determine the moment of inertia for the shaded area about the *y* axis.



iv. Determine the moment of inertia I_x of the shaded area about the y axis



v. Determine the moment of inertia of the area about the x axis